

UNIT-I

MICROWAVE TRANSMISSION LINES.

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INTRODUCTION:

Microwaves are electromagnetic waves whose frequencies range from 1 GHz to 1000 GHz ($1 \text{ GHz} = 10^9 \text{ Hz}$) to 10^{12} Hz . This corresponds to a range of wavelengths from 30 cm to 0.3 cm in free space.

- Because of very short wavelengths, microwaves are capable of almost freely propagating through the ionized layers in the atmosphere. This facilitates radio astronomic research of space and communication between the ground stations and space vehicles. —

The higher frequency edge of microwaves borders on the infrared and visible-light regions of spectrum. Hence microwaves behave more like rays of light than ordinary radio waves. Due to this unique behaviour, microwave frequencies are classified separately from radio waves.

The electromagnetic frequency spectrum is as shown :-

Microwave band							millimeter	submillimeter
L	S	C	X	KU	K	Ka		
1 GHz	2 GHz	4 GHz	8 GHz	12 GHz	18 GHz	40 GHz		0.300 THz
					27 GHz			

Microwave region in the electromagnetic spectrum, along with other frequency ranges as per CCIR recommendations is as shown:-

Band	FREQUENCY Range	Band Designation.
1	3 Hz - 30 Hz	Ultra low frequency (ULF)
2	30 Hz - 300 Hz	ELF
3	300 Hz - 3000 Hz (3 kHz)	VF
4	3 kHz - 30 kHz	VLF
5	30 kHz - 300 kHz	LF
6	300 kHz - 3000 kHz (3 MHz)	MF
7	3 MHz - 30 MHz	HF
8	30 MHz - 300 MHz	VHF
9	300 MHz - 3000 MHz (3 GHz)	UHF
10	3 GHz - 30 GHz	SHF
11	30 GHz - 300 GHz	EHF
12	300 GHz - 3000 GHz (3 THz)	Infra Red light
13	3 THz - 30 THz	Infra Red light
14	30 THz - 300 THz	Infra Red light
15	300 THz - 3000 THz (3 PHz)	Visible light
16	3 PHz - 30 PHz	Ultra violet light
17	30 PHz - 300 PHz	X-rays
18	300 PHz - 3000 PHz (3 EHz)	Gamma rays
19	3000 PHz - 30 EHz (3 EHz)	cosmic rays.

Standard band designations for microwave freq.
listed as per IEEE (Industry) standard.

BAND Designations.	Frequency Range (GHz).
UHF	0.3 - 3.0
L	1.1 - 1.7
LS	1.7 - 2.6
S	2.6 - 3.9
C	3.9 - 8.0
X	8.0 - 12.5
KU	12.5 - 18
K	18 - 26
Ka	26 - 40
Q	33 - 50
U	40 - 60
M	50 - 75
E	60 - 90
F	90 - 140
G	140 - 220
R	220 - 300
Sub millimeter	> 300

The New us/military microwave Frequency designation is as shown.

Band Designation	Frequency range (GHz)
A	0.1 - 0.25
B	0.25 - 0.5
C	0.5 - 1.0
D	1.0 - 2.0
E	2.0 - 3.0
F	3.0 - 4.0
G	4.0 - 6.0
H	6.0 - 8.0
I	8.0 - 10.0
J	10.0 - 20.0
K	20.0 - 40.0
L	40.0 - 60.0
M	60.0 - 100.0
N	100.0 - 140.0

ADVANTAGES OF MICROWAVES.

1. Increased Bandwidth Availability:-

Microwaves have large band widths ($1\text{GHz} - 1036\text{Hz}$). The advantage of large band widths is that the frequency range of information channels will be a small percentage of the carrier freq and more information can be transmitted in microwave frequency range.

2. Improved Directive Properties.

As frequency increases, directivity increases and beamwidth decreases. Hence the beamwidth of radiation θ is proportional to λ/D .

As frequency $\uparrow$, λ decreases hence power radiated and gain increases.

3. Fading effect and reliability:-

Fading effect due to variation in the transmission medium is more effective at low frequency. Due to "line of sight" (LoS) propagation and high frequencies there is less fading effect and hence microwave communication is more reliable.

4. Power Requirements :-

Tx/Rx power requirements are pretty low at microwave frequencies compared to that at short wave band.

5. Transparency property of microwaves :

microwave frequency band ranging from 300 MHz - 10 GHz are capable of freely propagating through the ionized layers surrounding the Earth as well as through the atmosphere.

microwave band facilitates the study of microwave radiation from the sun and stars in radio astronomical research of space.

APPLICATIONS :-

Microwaves have a broad range of applications in modern technologies. Most important among them are in long distance comm systems, radars, radio astronomy, navigation etc. Broadly the applications can be in the areas listed below:-

1. TELECOMMUNICATION :- Space Communication, (earth to space and space to earth), telemetry comm link for railways etc.

2. RADARS : Detect Aircraft, track/guide supersonic missiles, observe and track weather patterns, etc.

3. COMMERCIAL AND INDUSTRIAL APPLICATIONS use heat property of microwaves.

- i) Microwave (2.45 GHz, 600W)
- ii) Drying machines - textile, food and paper industry for drying clothes, etc.
- iii) Food processing industry - pre cooling / cooking, pasteurising / sterilising, heat-frozen / rehydrated pre cooled meats, roasting of food grains / beans.
- iv) Rubber industry / plastics / chemical / forest product industries.
- v) Mining / public works, breaking rock, tunnel boring etc.
- vi) Drying inks, drying / sterilising grains, drying / sterilising pharmaceuticals etc.
- vii) Biomedical applications - diathermy for localized superficial heating, deep electromagnetic heating for treatment of cancer, etc.

4. Electronic Warfare: ECM / ECCM (Electronic Counter measure / Electronic Counter counter measure) systems, spread spectrum systems.

5. Identifying objects or personnel by non contact method.

6. Light generated charge carriers in a microwave semiconductor makes it possible to create a whole new world of microwave devices, fast jitter-free switches, phase shifters, HF generation, tuning elements etc.

RECTANGULAR WAVEGUIDE MICROWAVE TX LINES:-

Microwaves propagate through various microwave circuits, components and devices that act as section of microwave transmission lines that are broadly called waveguides. The microwave signals can either be radiated into space via an antenna or can be transmitted over a transmission line. The conventional open wire transmission lines are not suitable for microwave transmission. The reason being, high radiation losses that are associated when the wavelength becomes smaller than the physical lengths of the conventional lines at high frequency.

At microwave frequencies, the following transmission lines will be employed -

i) Single conductor lines (waveguides)

- Rectangular waveguides
- Circular waveguides.
- Ridge waveguides.

ii) Multiconductor lines

- Coaxial lines
- Microstrip lines
- Strip lines
- coplanar lines etc.

iii) Open boundary structures

- Dielectric rods
- Open waveguides etc.

→ The single conductor lines support TE or TM wave while open conductor guides can support a combination of TE and TM modes. multiconductor lines normally support TEM or quasi TEM mode of propagation.

RECTANGULAR WAVEGUIDES :-

A Rectangular waveguide is a hollow metallic tube with a rectangular cross-section. The conducting walls of the guide confine the electromagnetic fields and thereby guide the electromagnetic wave. A number of distinct field configurations or modes can exist in waveguides. When the waves travel longitudinally down the guide, the plane waves are reflected from wall to wall.

This process results in a component of either electric or magnetic field in the direction of propagation of the resultant wave; therefore the wave is no longer a transverse electromagnetic (TEM) wave. Fig (1) below shows that any uniform plane wave in a lossless guide may be resolved into TE and TM waves.

When the wavelength λ is in the direction of propagation of the incident wave, there will be one component λ_n in the direction normal to the reflecting plane and another λ_p parallel to the plane. These components are

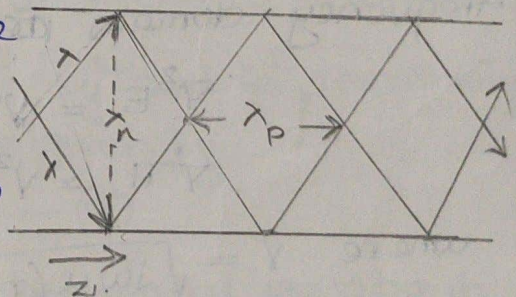


Fig 1) - plane wave reflected in a waveguide.

$$\lambda_n = \frac{\lambda}{\cos \theta} \quad (1); \quad \lambda_p = \frac{\lambda}{\sin \theta} \quad (2)$$

A plane wave in a waveguide resolves into two components: one standing wave in the direction normal to the reflecting walls of the guide and one travelling wave in the direction parallel to the reflecting walls. In lossless waveguides the modes may be classified as either TE or TM mode. In rectangular guides the modes are designated TE_{mn} or TM_{mn} .

The integer m denotes the number of half waves of electric or magnetic intensity in the x direction, and n is the number of half waves in the y direction if the propagation of the wave is assumed in the positive z direction.

SOLUTION OF WAVE EQUATIONS IN RECTANGULAR co-ordinates

A rectangular co-ordinate system is shown in fig 2.

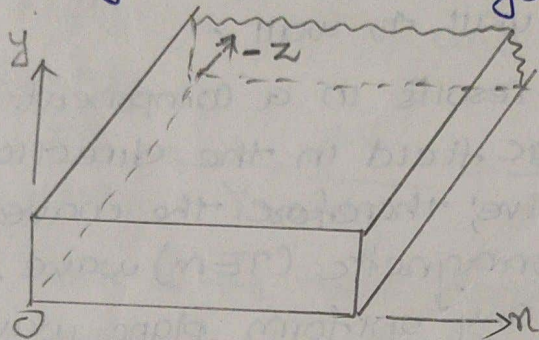


Fig 2. Rectangular coordinates

The electric and magnetic wave equations in frequency domain ~~are~~ is given by,

$$\nabla^2 E = \gamma^2 E \quad \text{--- (3)}$$

$$\nabla^2 H = \gamma^2 H \quad \text{--- (4)}$$

where $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$. These are called the vector wave equations.

Rectangular coordinates are the usual right hand system. The rectangular components of E or H satisfy the complex scalar wave equation or Helmholtz equation.

$$\nabla^2 \psi = \gamma^2 \psi \quad \text{--- (5)}$$

The Helmholtz equation in rectangular co-ordinates

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \gamma^2 \psi \quad \text{--- (6)}$$

This is a linear and inhomogeneous partial differential equation in three dimensions. By the method of separation of variables, the solution is assumed in the form of,

$$\psi = X(x) Y(y) Z(z) \quad \text{--- (7)}$$

where, $X(x)$ = a function of the x co-ordinate only
 $Y(y)$ = a function of the y coordinate only,
 $Z(z)$ = a function of the z coordinate only.

Substitute eq (7) in eq (6) and divide the resultant by eq (7) yields,

$$\frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} + \frac{1}{Z} \frac{d^2 Z}{dz^2} = \gamma^2 \quad \text{--- (8)}$$

Since the sum of the three terms on the left hand side is a constant and each term is independently variable, it follows that each term must be equal to a constant.

Let the three terms be k_x^2 , k_y^2 and k_z^2 , respectively, then the separation equation is given by

$$-k_x^2 - k_y^2 - k_z^2 = \gamma^2 \quad \text{--- (9)}$$

The general solution of each differential equation in equation (8),

$$\frac{d^2 X}{dx^2} = -k_x^2 X \quad \text{--- (10)}$$

$$\frac{d^2 Y}{dy^2} = -k_y^2 Y \quad \text{--- (11)}$$

$$\frac{d^2 Z}{dz^2} = -k_z^2 Z \quad \text{--- (12)}$$

will be in the form of

$$X = A \sin(k_x x) + B \cos(k_x x) \quad \text{--- (13)}$$

$$Y = C \sin(k_y y) + D \cos(k_y y) \quad \text{--- (14)}$$

$$Z = E \sin(k_z z) + F \cos(k_z z) \quad \text{--- (15)}$$

The total solution of the Helmholtz equation in rectangular coordinates is,

$$\psi = [A \sin(k_x x) + B \cos(k_y y)] [C \sin(k_y y) + D \cos(k_y y)] \\ \times [E \sin(k_z z) + F \cos(k_z z)] \quad \text{--- (16)}$$

The propagation of the wave in the guide is conventionally assumed in the positive z direction. It should be noted that the propagation constant γ_g in the guide differs from the intrinsic propagation γ of the dielectric.

Let,

$$\gamma_g^2 = \gamma^2 + k_x^2 + k_y^2 = \gamma^2 + k_c^2 \quad \text{--- (17)}$$

where $k_c = \sqrt{k_x^2 + k_y^2}$ is usually called the cut off wave number. For a lossless dielectric $\gamma^2 = -\omega^2 \mu \epsilon$, then

$$\gamma_g = \pm \sqrt{\omega^2 \mu \epsilon - k_c^2} \quad \text{--- (18)}$$

There are three cases for the propagation constant γ_g in the wave guide.

Case I. There will be no wave propagation in the guide, if $\omega_c^2 \mu \epsilon = k_c^2$ and $\gamma_g = 0$. This is the critical condition for cut off propagation. The cut off frequency is expressed as,

$$f_c = \frac{1}{2\pi \sqrt{\mu \epsilon}} \sqrt{k_x^2 + k_y^2} \quad \text{--- (19)}$$

Case II. The wave will be propagating in the guide if $\omega^2 \mu \epsilon > k_c^2$ and,

$$\gamma_g = \pm j\beta_g = \pm j\omega \sqrt{\mu \epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad \text{--- (20)}$$

This means that the operating frequency must be above the cut off frequency in order for a wave to propagate in the guide.

Case - II) The wave will be attenuated if $\omega^2 \mu \epsilon < k_c^2$ and $\gamma_g = \pm \alpha_g = \pm \omega \sqrt{\mu \epsilon} \sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$ — (21).

This means that if the operating frequency is below the cut off frequency, the wave will decay exponentially with respect to a factor of $-\alpha_g z$ and there will be no wave propagation because the propagation constant is a real quantity. Therefore the solution to the Helmholtz equation in rectangular co-ordinates is given by,

$$\psi = [A \sin(k_x x) + B \cos(k_x x)] [C \sin(k_y y) + D \cos(k_y y)] e^{-\gamma_g z} \quad \text{--- (22)}$$

TE MODES IN RECTANGULAR WAVEGUIDES.

It is assumed that the waves are propagating in the positive z direction in the waveguide. Fig (3) shows the co-ordinates of a rectangular waveguide.

The TE_{mn} modes in a rectangular waveguide are characterized by $E_z = 0$. In other words, the z component of the magnetic field, H_z , must exist in order to have energy transmission in the guide.

Consequently, from a given Helmholtz equation,

$$\nabla^2 H_z = \gamma^2 H_z \quad \text{--- (23)}$$

a solution in the form of

$$H_z = \left[A_m \sin\left(\frac{m\pi x}{a}\right) + B_m \cos\left(\frac{m\pi x}{a}\right) \right] \times \left[C_n \sin\left(\frac{n\pi y}{b}\right) + D_n \cos\left(\frac{n\pi y}{b}\right) \right] e^{-\gamma_g z} \quad \text{--- (24)}$$

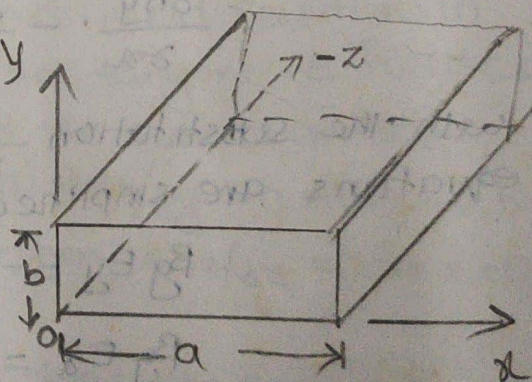


Fig. Co-ordinates of a rectangular guide.

will be determined in accordance with the given boundary conditions, where $k_x = m\pi/a$ and $k_y = n\pi/b$ are replaced. For a lossless dielectric, Maxwell's curl equations in frequency domain are,

$$\nabla \times E = -j\omega\mu H \quad (25)$$

$$\nabla \times H = j\omega\epsilon E \quad (26)$$

In rectangular co-ordinates, their components are

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \quad (27)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad (28)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad (29)$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x \quad (30)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \quad (31)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \quad (32)$$

with the substitution $\frac{\partial}{\partial z} = -j\beta_g$ and $E_z = 0$, the foregoing equations are simplified to

$$\beta_g E_y = -\omega\mu H_x \quad (33)$$

$$\beta_g E_x = \omega\mu H_y \quad (34)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = j\omega\mu H_z \quad (35)$$

$$\frac{\partial H_z}{\partial y} + j\beta_g H_y = j\omega\epsilon E_x \quad (36)$$

$$-j\beta_g H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \quad (37)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0 \quad (38)$$

Solving these six equations for E_x , E_y , H_x and H_y in terms of H_z will give the TE-mode field equations in rectangular waveguides as

$$E_x = -\frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y} \quad \text{--- (39)}$$

$$E_y = \frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x} \quad \text{--- (40)}$$

$$E_z = 0 \quad \text{--- (41)}$$

$$H_x = -\frac{j\beta_g}{k_c^2} \frac{\partial H_z}{\partial x} \quad \text{--- (42)}$$

$$H_y = -\frac{j\beta_g}{k_c^2} \frac{\partial H_z}{\partial y} \quad \text{--- (43)}$$

$$H_z = \sum q, \text{ (44)} \quad \text{--- (44)}$$

where $k_c^2 = \omega^2\epsilon\mu - \beta_g^2$ has been replaced

Differentiating eq (24) with respect to x and y and then substituting the results in eq 39 to 43, yield a set of field equations. The boundary conditions are applied to the newly found field equations in such a manner that either the tangent E field or the normal H field vanishes at the surface of the conductor, since $E_x = 0$, then $\partial H_z / \partial y = 0$ at $y = 0, b$, hence $C_n = 0$. Since $E_y = 0$, then $\partial H_z / \partial x = 0$ at $x = 0, a$. Hence $A_m = 0$.

It is generally concluded that the normal derivative of H_z must vanish at the conducting surfaces—that is

$$\frac{\partial H_z}{\partial n} = 0 \quad \text{--- (45)}$$

at the guide walls. Therefore the magnetic field in the positive z direction is given by

$$H_z = H_{0z} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad \text{--- (46)}$$

where H_{0z} is the amplitude constant.

Substitution of eq (46) in eq. (39) through (43) yields the TEMn field equations in rectangular waveguides as,

$$E_x = E_{0x} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (47)$$

$$E_y = E_{0y} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (48)$$

$$E_z = 0 \quad (49)$$

$$H_x = H_{0x} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (50)$$

$$H_y = H_{0y} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (51)$$

$$H_z = 0 \quad (52)$$

where $m = 0, 1, 2, \dots$

$n = 0, 1, 2, \dots$

$m = n = 0$ expected.

The cutoff wave number k_c , as defined by eq (17) for the TEMn modes, is given by,

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \omega_c \sqrt{\mu\epsilon} \quad (53)$$

where a and b are in meters. The cut off frequency, as defined in eq (19) for the TEMn modes, is

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}} \quad (54)$$

The propagation constant (or the phase constant here) β_g , as defined in eq (18) is expressed by

$$\beta_g = \omega \sqrt{\mu\epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad (55)$$

The phase velocity in the positive z direction for the TEMn modes is shown as

$$V_g = \frac{\omega}{\beta_g} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}} \quad (56)$$

(9)

where $V_p = 1/\sqrt{\mu\epsilon}$ is the Phase velocity in an unbounded dielectric.

The characteristic wave Impedance of TE_{mn} modes in the guide can be derived from eq (33) and (24):

$$Z_g = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta_g} = \frac{\eta}{\sqrt{1-(f_c/f)^2}} \quad (57)$$

where $\lambda = V_p/f$ is the wavelength in an unbounded dielectric.

Since the cut off frequency shown in eq (54) is a function of the modes and guide dimensions, the physical size of the waveguides will determine the propagation of the modes.

→ Whereas Dominant and Degenerate modes:-

Whenever two or more modes have the same cut off frequency, they are said to be degenerate modes. In a rectangular guide the corresponding TE_{mn} and TM_{mn} modes are always degenerate.

In a square guide the TE_{mn} , TE_{nm} , TM_{mn} and TM_{nm} modes ~~are always degenerate~~ form a four some of degeneracy.

→ The mode with the lowest cut off frequency in a particular guide is called the Dominant mode. The dominant mode in a rectangular guide with $a > b$ is the TE_{10} mode. Each mode has a specific mode pattern (or field pattern).

It is normal for all modes to exist simultaneously in a given waveguide. However, only the dominant mode propagates, and the highest modes near the sources or discontinuities decay very fast.

TM MODES IN RECTANGULAR WAVEGUIDES

The TM_{mn} modes in a Rectangular waveguide are characterized by $H_z = 0$. In other words, the z component of an electric field E must exist in order to have energy transmission in the guide. Consequently the Helmholtz equation for E in the rectangular coordinates is given by,

$$\nabla^2 E_z = \gamma^2 E_z \quad \text{--- (59)}$$

A solution of the Helmholtz equation is in the form of

$$E_z = \left[A_m \sin\left(\frac{m\pi x}{a}\right) + B_m \cos\left(\frac{m\pi x}{a}\right) \right] \left[C_n \sin\left(\frac{n\pi y}{b}\right) + D_n \cos\left(\frac{n\pi y}{b}\right) \right] e^{-\gamma_g z} \quad \text{--- (60)}$$

which must be determined according to the given boundary conditions. The procedures for doing so are similar to those used in finding the TE-mode wave.

The boundary conditions on E_z require that the field vanishes at the waveguide's wall, since the tangent components of the electric field E_z is zero on the conducting surface. This requirement is that for $E_z = 0$ at $x=0, a$, then $B_m = 0$, and for $E_z = 0$ at $y=0, b$, then $D_n = 0$. Thus the solution as shown in eq (60) reduces to,

$$E_z = E_0 z \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma_g z} \quad \text{--- (61)}$$

where $m = 1, 2, 3, \dots$

$n = 1, 2, 3, \dots$

If either $m=0$ or $n=0$, the field intensities all vanish. So there is no TM_{01} or TM_{10} mode in a rectangular waveguide, which means that TE_{10} is the dominant mode in a rectangular waveguide for $a > b$. For $H_z = 0$, the field equations, after expanding $\nabla \times H = j\omega \epsilon E$

are given by

$$\frac{\partial E_z}{\partial y} + j\beta_g E_y = -j\omega\mu H_x \quad (62)$$

$$j\beta_g E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y \quad (63)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0 \quad (64)$$

$$\beta_g H_y = \omega\epsilon E_x \quad (65)$$

$$-\beta_g H_x = \omega\epsilon E_y \quad (66)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \quad (67)$$

These equations can be solved simultaneously for E_x , E_y , H_x , and H_y in terms of E_z .

The resultant field equations for TM modes are

$$E_x = \frac{-j\beta_g}{k_c^2} \frac{\partial E_z}{\partial x} \quad (68)$$

$$E_y = \frac{-j\beta_g}{k_c^2} \frac{\partial E_z}{\partial y} \quad (69)$$

$$E_z = \sum q G_1 \quad (70)$$

$$H_x = \frac{j\omega\epsilon}{k_c^2} \frac{\partial E_z}{\partial y} \quad (71)$$

$$H_y = \frac{-j\omega\epsilon}{k_c^2} \frac{\partial E_z}{\partial x} \quad (72)$$

$$H_z = 0 \quad (73)$$

where $\beta_g^2 = \omega^2\mu\epsilon = -k_c^2$ is replaced

Differentiating eq (61) with respect to x and y and substituting the results in eq (68) through (73) yield a new set of field equations. The TM_{mn} mode field equation in rectangular wave guides are

$$E_x = E_{0x} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (74)$$

$$E_y = E_{0y} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (75)$$

$$E_z = 0 \quad (76)$$

$$H_x = H_{0x} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (77)$$

$$H_y = H_{0y} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad (78)$$

$$H_z = 0 \quad (79)$$

Some of the TM modes characteristic equations are identical to those of the TE modes, but some are different. For convenience, all are shown here:

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}} \quad (80)$$

$$\beta_g = \omega\sqrt{\mu\epsilon} \sqrt{1 - (f_c/f)^2} \quad (81)$$

wavelength $\lambda_g = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}} \quad (82)$

Phase ϕ group velocity $v_g = \frac{v_p}{\sqrt{1 - (f_c/f)^2}} \quad (83)$

Impedance $z_g = \frac{\beta_g}{\omega\epsilon} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad (84)$

In practice, the rectangular-cavity resonator, circular cavity resonator and reentrant-cavity resonator are commonly used in many microwave applications.

A given resonator has an infinite number of resonant modes, and each mode corresponds to a definite resonant frequency. When the frequency of an impressed signal is equal to a resonant frequency, a maximum amplitude of the standing wave occurs, and the peak energies stored in the electric & magnetic fields are equal.

The mode having the lowest resonant frequency is known as the dominant mode.

RECTANGULAR CAVITY RESONATOR.

The electromagnetic field inside the cavity should satisfy Maxwell's equations, subject to the boundary conditions that the electric field is tangential to and the magnetic field normal to the metal walls must vanish. The geometry of a rectangular cavity is shown below in fig 1.

The wave equations in the rectangular resonator should satisfy the boundary condition of the zero tangential E at four of the walls. It is merely necessary to choose the harmonic functions in z to

satisfy this condition at the remaining two ~~and~~ end walls. These functions can be found if,

$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right) \quad (TE_{mnp}) \quad \text{--- (1)}$$

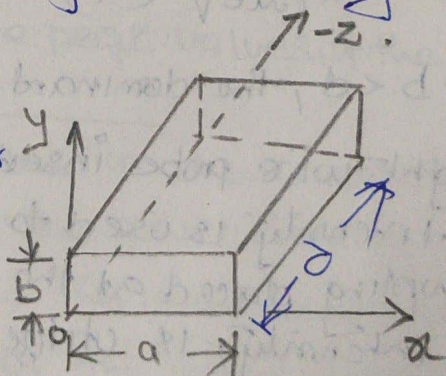


Fig. Co-ordinates of a rectangular cavity.

MICROWAVE TRANSMISSION LINES - II.RECTANGULAR GUIDES:-POWER TRANSMISSION IN RECTANGULAR WAVEGUIDES.

The power transmitted through a waveguide and the power loss in the guide walls can be calculated by means of the complex Poynting theorem. It is assumed that the guide is terminated in such a way that there is no reflection from the receiving end or that the guide is infinitely long compared with the wave length. From the Poynting theorem, the power transmitted through a guide is given by,

$$P_{tr} = \oint P \cdot ds = \oint \frac{1}{2} (E \times H^*) \cdot ds \quad - (1)$$

For a lossless dielectric, the time-average power flow through a rectangular guide is given by,

$$P_{av} = \frac{1}{2Z_g} \int_a |E|^2 da = \frac{Z_g}{2} \int_a |H|^2 da \quad - (2)$$

$$\text{where } Z_g = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

$$|E|^2 = |E_x|^2 + |E_y|^2$$

$$|H|^2 = |H_x|^2 + |H_y|^2$$

For TE_{mn} modes, the average power transmitted through a rectangular waveguide is given by,

$$P_{tr} = \frac{\sqrt{1 - (f_c/f)^2}}{2\eta} \int_0^b \int_0^a (|E_x|^2 + |E_y|^2) dx dy \quad - (3)$$

For TM_{mn} modes, the average power transmitted through a rectangular waveguide is given by,

$$P_{\text{loss}} = \frac{1}{2\eta\sqrt{1-(f_c/f)^2}} \int_0^b \int_0^a (|E_x|^2 + |E_y|^2) dx dy \quad \text{--- (4)}$$

where $\eta = \sqrt{\mu/\epsilon}$ is the intrinsic impedance in an unbounded dielectric.

→ POWER LOSSES IN RECTANGULAR WAVEGUIDES.

There are two types of power losses in a rectangular wave guide :-

1. losses in the dielectric
2. Losses in the guide wall.

Consider power losses caused by dielectric attenuation. In a low loss dielectric (that is $\sigma \ll \omega\epsilon$), the propagation constant for a plane wave travelling in an unbounded lossy dielectric is given by

$$\alpha = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} = \eta \frac{\sigma}{2} \quad \text{--- (1) as per above eq.}$$

The attenuation caused by the low loss dielectric dielectric in the rectangular waveguide for the TE_{mn} & TM_{mn} modes is given by,

$$\alpha_g = \frac{\sigma \eta}{2\sqrt{1-(f_c/f)^2}} \quad \text{for TE mode --- (2a)}$$

$$\alpha_g = \frac{\sigma \eta}{2} \sqrt{1-(f_c/f)^2} \quad \text{for TM mode --- (2b)}$$

As $f \gg f_c$, the attenuation constant in the guide approaches that for the unbounded dielectric given by eq (1). However if the operating freq is way below

the cut off frequency, $f < f_c$, the attenuation constant becomes very large and non propagation occurs.

2 → Consider power losses caused by the Guide walls.

When the electric and magnetic intensities propagate through a lossy waveguide, their magnitudes may be written,

$$|E| = |E_0z| e^{-\alpha_g z} \quad \text{--- (3)}$$

$$|H| = |H_0z| e^{-\alpha_g z} \quad \text{--- (4)}$$

where E_0z and H_0z are the field intensities at $z=0$.

It is interesting to note that for a low loss guide, the time-average power flow decreases proportionally to $e^{-2\alpha_g z}$. Hence,

$$P_{tx} = (P_{tx} + P_{loss}) e^{-2\alpha_g z} \quad \text{--- (5)}$$

for $P_{loss} \ll P_{tx}$ and $2\alpha_g z \ll 1$,

$$\frac{P_{loss}}{P_{tx}} + 1 = 1 + 2\alpha_g z \quad \text{--- (6)}$$

Finally,

$$\alpha_g = \frac{P_L}{2P_{tx}} \quad \text{--- (7)}$$

where P_L is the power loss per length. Consequently, the attenuation constant of the guide walls is equal to the ratio of the power loss per unit length to twice the power transmission transmitted through the guide.

Since the electric and magnetic field intensities established at the surface of a low loss guide wall decay exponentially with respect to the skin depth while the waves progress into the walls, it is better to define a surface resistance of the guide walls

$$\text{as, } R_s = \frac{\rho}{\delta} = \frac{1}{\sigma \delta} = \frac{\alpha_g}{\sigma} = \sqrt{\frac{\pi f \mu}{\sigma}} \text{ } \Omega/\text{square} \quad (8)$$

where ρ = resistivity of the conducting wall in Ω -meter
 σ = Conductivity in mhos per meter
 δ = Skin depth or depth of penetration in meters

The power loss per unit length of guide is obtained by integrating the power density over the surface of the conductor corresponding to the unit length of the guide, this is,

$$P_L = \frac{R_s}{2} \int_S |H_t|^2 ds \text{ W/unit length} \quad (9)$$

where H_t is the tangential component of magnetic intensity at the guide walls.

substitution of eq (2) of (power transmission) and (9) above yields,

$$\alpha_g = \frac{R_s \int_S |H_t|^2 ds}{2Z_g \int_a |H|^2 da} \quad (10)$$

where,

$$|H|^2 = |H_z|^2 + |H_t|^2 \quad (11)$$

$$|H_t|^2 = |H_{tx}|^2 + |H_{ty}|^2 \quad (12)$$

IMPOSSIBILITY OF TEM MODE:-

3

& For a TEM mode, $E_z = 0$ and $H_z = 0$. Substituting these values in field equations along x and y directions, E_x, E_y, H_x, H_y vanishes and hence a TEM wave cannot exist inside a waveguide.

Hence, multi conductor lines normally support TEM or quasi TEM mode of propagation. The single conductor lines support TE or TM waves.

TEM mode cannot exist in a hollow waveguide. It can exist only in the 2 conductor system where the centre conductor does not exist.

→ The walls of waveguide are perfect conductors so a potential cannot exist across them. Thus any electric field would be short circuited by the walls. So, TEM wave would not propagate in waveguide.

The transverse electric and transverse magnetic (TEM) modes as transmission line modes are characterized by $E_z = H_z = 0$.

This means that the electric and magnetic fields are completely transverse to the direction of wave propagation. This mode cannot exist in hollow waveguides, since it requires two conductors such as the coaxial transmission line and two open-wire line.

STRIP LINES :-

Striplines are essentially modifications of the two wire lines and coaxial lines. These are basically planar transmission lines, that are widely used at frequencies from 100 MHz to 100 GHz. Fig (a) below shows a cross-sectional view of the stripline structures.

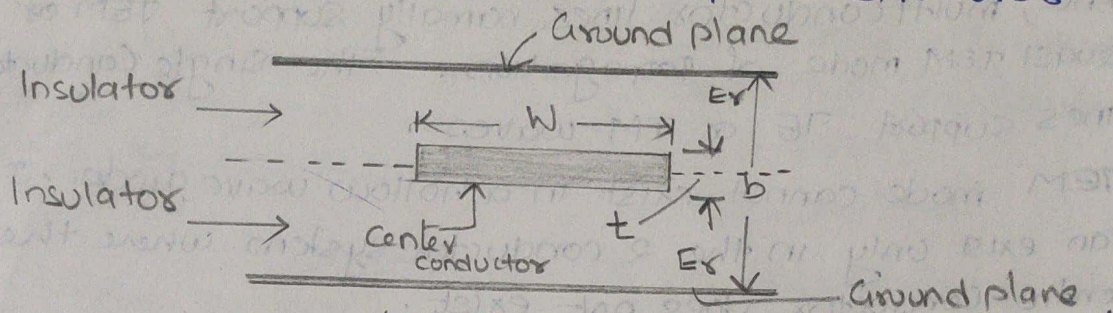


Fig (a) - Stripline transmission line.

Fig (2), shows a strip line consists of a central thin conducting strip of width 'w' which is greater than its thickness 't', placed inside the low loss dielectric (ϵ_r) substrate of thickness $b/2$ between two wide ground plates. Usually the thicknesses of the metallic central conductor and the metallic ground plates are the same. The dominant mode for the strip line is a TEM mode shown in Fig (3) and the fields are confined within the transmission line with no radiation losses. The width of the ground planes is at least 5 times greater than the spacing between the plates there by avoiding any vertical side walls at the two transverse ends.

There are practically no fringing fields after a certain distance from the edges of the center conductor.

for $b < \lambda/2$, there will be no propagation in the transverse region.

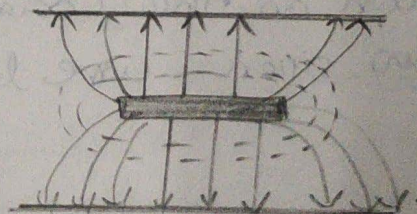


Fig. TEM mode of Stripline

— electric field lines.
--- magnetic field lines

DISADVANTAGES OF STRIPLINE

1. The circuit of stripline is not accessible during development for adjustment and tuning and also it is difficult.
2. It is difficult to mount discrete and active components (like transistors, diodes, circulators, chip resistors).
3. Interconnectivity is not possible.

MICROSTRIP LINES. — INTRODUCTION.

Microstrip line is an unsymmetrical stripline that is nothing but a parallel plate transmission line having dielectric substrate, the one face of which is metallised ground and the other face has a thin conducting strip of certain width ' w ' and thickness ' t '. This is shown in Fig (4) below. The top ground plane is not present in a microstrip as compared to a stripline. Sometimes a cover plate is used for shielding purposes but it is kept much farther away than the ground plane so as not to affect the microstrip field lines as in Fig (5).

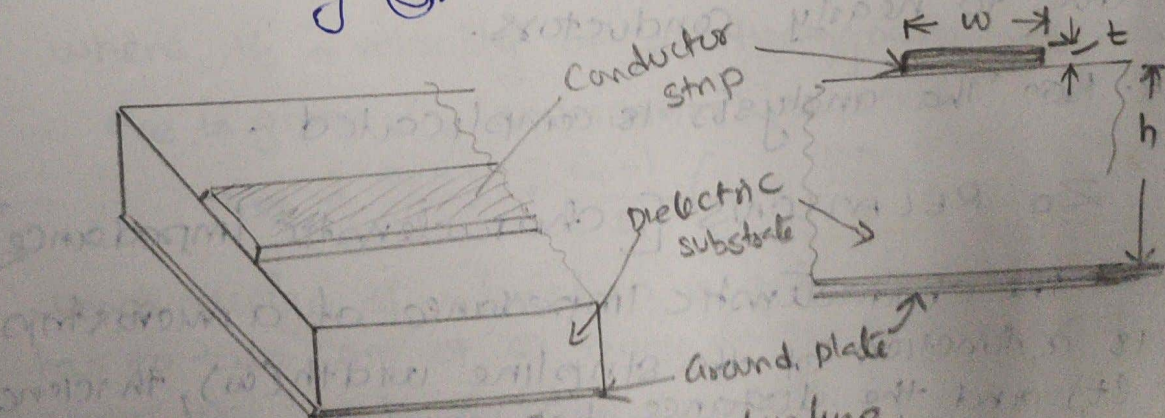


Fig (4) microstrip line.

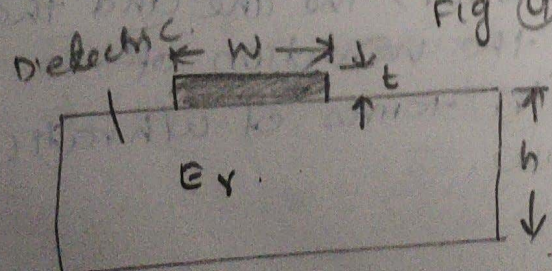


Fig (5) microstrip line with a cover plate.

Advantages of microstrip line over strip lines, coaxial line and wave guides:-

1. Complete conductor pattern may be deposited and processed on a single dielectric substrate which is supported by a single metal ground plane. Thus fabrication costs would be substantially lower than strip line, coaxial or waveguide circuits.
2. Due to the plane nature of the ^{micro}strip line structure both packaged and unpackaged semiconductor chips can be conveniently attached to the microstrip element.
3. ~~At~~ There is an easy access to the top surface making it easy to mount passive or active discrete devices, and also for making minor adjustments after the circuit has been fabricated.

Limitations of microstrip lines.

1. Due to the openness of the microstrip structure, they have higher radiation losses or interference due to nearby conductors.
2. ~~But~~ The analysis is complicated.

Z₀ RELATIONS [characteristic Impedance]

The characteristic impedance of a microstrip is a function of the stripline width (w), thickness (t) and the distance between the line and the ground plane (h). In fact, the variation of characteristic impedance in terms of w/h ratio is shown in fig (5) below.

Empirical relation for Z_0 for a microstrip line is given by,

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln\left(\frac{4h}{d}\right) \text{ for } h \gg d. \quad \text{--- (1)}$$

where,

ϵ_r = dielectric constant of the dielectric medium.

h = distance between the microstrip line and the ground plane

d = diameter of the wire.

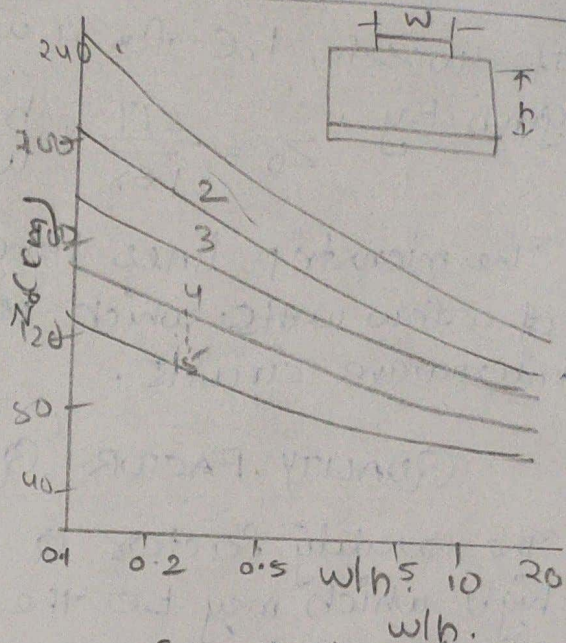


Fig Z_0 vs w/h ratios

Since the cross-section of microstrip line is rectangular diameter (d) also has an empirical relation given by

$$d = 0.67w \left(0.8 + \frac{t}{w}\right) \quad \text{--- (2)}$$

The phase velocity of a microstrip line is given by,

$$V_p = V_c / \sqrt{\epsilon_{re}} \quad \text{--- (3)}$$

where V_c = velocity of electromagnetic waves

ϵ_{re} is given by an empirical relation

$$\epsilon_{re} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(1 + \frac{10h}{w}\right)^{-1/2} \quad \text{--- (4)}$$

Hence design of microstrip is quite complex as it has to take care of so many factors discussed above like $w, h, \epsilon_r, \epsilon_{re}, \epsilon_{eff}$ etc.

Taking into account the relationships for ϵ_{re} and d , (from eq 3 and 4) Z_0 can be written as,

$$Z_0 = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln \left[\frac{5.98h}{0.8w + t} \right] \text{ for } h < 0.8w \quad \text{--- (5)}$$

If $w \gg h$, i.e. for a wide microstrip line, Z_0 is given by, $Z_0 = \frac{377}{\sqrt{\epsilon_r}} \cdot \frac{h}{w}$. — (6)

The microstrip lines have a power handling capacity of a few watts which is quite adequate for most microwave circuits.

QUALITY FACTOR 'Q' of microstrip lines.

The quality factor Q of a microstrip line is very high which may be the requirement for high quality resonant mics. It is however limited by the radiation losses of the substrate and with low dielectric constant.

The Q of a microstrip line is ~~very high which may be the requirement~~ given by,

$$Q_d = \frac{1}{\tan \delta} \quad \text{--- (7)}$$

where δ = dielectric loss tangent.

For a uniform current distribution in the microstrip line, the ohmic attenuation constant of a wide microstrip line is given by,

$$\alpha_c = \frac{8.686 R_s}{Z_0 w} \quad \text{dB/cm}$$

And that the characteristic impedance of a wide microstrip line, (as per eq 6 above) is,

$$Z_0 = \frac{h}{w} \sqrt{\frac{\mu}{\epsilon}} = \frac{377}{\sqrt{\epsilon_r}} \frac{h}{w} \quad \Omega.$$

The wavelength in the microstrip line is

$$\lambda_g = \frac{30}{f \sqrt{\epsilon_r}} \quad \text{cm}.$$

PROBLEMS :- UNIT - II

1Q. Calculate the resonant frequency of a circular resonator of following dimensions. Diameter = 12.5 cms and length = 5 cms for TM_{012} mode.

Sol. Diameter = $2a = 12.5$ cms.

$$a = 6.25 \text{ cms}, l = d = 5 \text{ cms.}$$

Here $n = 0, m = 1, p = 2$

$$f_0 = \frac{c}{2\pi} \left[\left(\frac{P_{nm}}{a} \right)^2 + \left(\frac{p\pi}{d} \right)^2 \right]^{1/2}$$

$$P_{nm} = P_{01} = 2.405 \text{ (dominant mode) } TM_{01}.$$

$$f_0 = \frac{3 \times 10^{10}}{2\pi} \left[\left(\frac{2.405}{6.25} \right)^2 + \left(\frac{2\pi}{5} \right)^2 \right]^{1/2} = 6.27 \text{ GHz.}$$

2Q. Calculate the lowest resonant frequency of a rectangular cavity resonator of dimension $a = 2$ cms, $b = 1$ cm, $d = 3$ cms.

Sol. Lowest resonant frequency is obtained for the dominant mode TE_{101} [$f = c/\lambda$, $\lambda \uparrow$ as $f \downarrow$, In dominant mode λ_c is maximum].

$\therefore$ The dominant mode is TE_{101} , $m = 1, n = 0$ and $p = 1$

$$f_0 = \frac{c}{2} \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 + \left(\frac{p}{d} \right)^2 \right]^{1/2}$$

$$f_0 = \frac{3 \times 10^{10}}{2} \left[\left(\frac{1}{2} \right)^2 + 0 + \left(\frac{1}{3} \right)^2 \right]^{1/2}$$

$$= 9 \text{ GHz.}$$

3Q. Calculate the resonant frequency of a circular resonator of dimensions $a = 3$ cms, $b = 2$ cms, and $l = 4$ cms when the mode of operation is TE_{101} .

Sol. $f_0 = \frac{c}{2} \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 + \left(\frac{p}{d} \right)^2 \right]^{1/2}$

$$= \frac{3 \times 10^{10}}{2} \left[\left(\frac{1}{3} \right)^2 + \left(\frac{1}{4} \right)^2 \right]^{1/2} = \underline{\underline{6.25 \text{ GHz}}}$$

④. A certain microstrip line has the following parameters :

$\epsilon_r = 5.23$, $h = 7 \text{ mils}$, $t = 2.8 \text{ mils}$, $w = 10 \text{ mils}$
 calculate the characteristic impedance Z_0 of the line :

$$\begin{aligned} \underline{\underline{\text{Sol}}}\quad Z_0 &= \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln \left[\frac{5.98h}{0.8w + t} \right] \\ &= \frac{87}{\sqrt{5.23 + 1.41}} \ln \left[\frac{5.98 \times 7}{0.8 \times 10 + 2.8} \right] \\ &= \underline{\underline{45.78 \Omega}} \end{aligned}$$

① Calculate a few of the resonant frequencies of a cavity made from a section of a rectangular waveguide 2cm by 1cm by 3cm long. Identify the mode of oscillation in the cavity appropriate to each frequency.

sol Resonant frequency of the rectangular cavity:

$$f_0 = \frac{v_0}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$= \frac{3 \times 10^8}{2} \sqrt{\left(\frac{m}{0.02}\right)^2 + \left(\frac{n}{0.01}\right)^2 + \left(\frac{p}{0.03}\right)^2} \quad \begin{array}{l} a = 2\text{cm} \\ b = 1\text{cm} \\ c = 3\text{cm} \end{array}$$

$$\therefore f_0 = 0.25 \times 10^{10} \sqrt{9m^2 + 36n^2 + 4p^2}$$

for TE_{101} mode: $f_0 = 0.25 \times 10^{10} \sqrt{13} = 9\text{GHz}$, $m=1, n=0, p=1$

for TE_{011} mode: $f_0 = 0.25 \times 10^{10} \sqrt{40} = 15.8\text{GHz}$, $m=0, n=1, p=1$

for TE_{102} mode: $f_0 = 0.25 \times 10^{10} \sqrt{25} = 12.5\text{GHz}$

for TE_{201} mode: $f_0 = 0.25 \times 10^{10} \sqrt{40} = 15.8\text{GHz}$

for TE_{111} mode: $f_0 = 0.25 \times 10^{10} \sqrt{49} = 17.5\text{GHz}$

for TM_{111} mode: $f_0 = 0.25 \times 10^{10} \sqrt{49} = 17.5\text{GHz}$

②. A cavity resonator with dimensions $a=2\text{cm}$, $b=1\text{cm}$ is excited by TE_{101} mode of 20GHz . Calculate the length of the cavity.

sol for the dominant mode, TE_{101} , $m=1, n=0, p=1$.

$$f_0 = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}$$

$$= \frac{v_0}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$20 \times 10^9 = \frac{3 \times 10^{10}}{2} \sqrt{\left(\frac{1}{2}\right)^2 + 0 + \left(\frac{1}{a}\right)^2}$$

$$\frac{16}{9} = \frac{1}{4} + \frac{1}{a^2}$$

$$\frac{1}{a^2} = 1.78 - 0.25 = 1.53 \Rightarrow a = \underline{\underline{0.8 \text{ cm}}}$$

③ A cavity resonator in the form of a cube is to operate in the TE_{101} mode. If the side of the cube is 10 cm, calculate the resonant frequency.

Sol $f_0 = \frac{V_0}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$
 $= \frac{3 \times 10^{10}}{2} \sqrt{\left(\frac{1}{10}\right)^2 + 0 + \left(\frac{1}{10}\right)^2}$ for a cube, $a = b = d = 10 \text{ cm}$,
 the equation becomes,

$$f_0 = \frac{V_0}{2a} \sqrt{m^2 + n^2 + p^2}$$

So, f_0 is same for TE_{101} , TE_{011} , TM_{110} as the resonant wavelength is equal to the diagonal of the cuboid.

$$\therefore f_0 = \frac{1.5 \times 10^8 \times \sqrt{2}}{0.1} = \underline{\underline{2.12 \text{ GHz}}}$$

④ A certain microstrip line has the following parameters
 $\epsilon_r = 6.0$, $h = 7.5 \text{ cm}$, $t = 2.8 \text{ cm}$, $w = 9.5 \text{ cm}$.
 Calculate the characteristic impedance of the line.

$$Z_0 = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln \left[\frac{5.98h}{0.8w + t} \right] \text{ for } \boxed{h < 0.8w}$$

$$= \frac{87}{\sqrt{6 + 1.41}} \ln \left[\frac{5.98 \times 7.5}{0.8 \times 9.5 + 2.8} \right]$$

$$= \underline{\underline{46.71 \Omega}}$$

$\Rightarrow \frac{377}{\sqrt{\epsilon_r + 1.41}} \ln \left[\frac{5.98h}{0.8w + t} \right]$

CAVITY RESONATORS.

A cavity resonator is a metallic enclosure that confines the electromagnetic energy.

When one end of the wave guide is terminated in a shorting plate, there will be reflections and hence standing waves as shown in Fig (1) below.

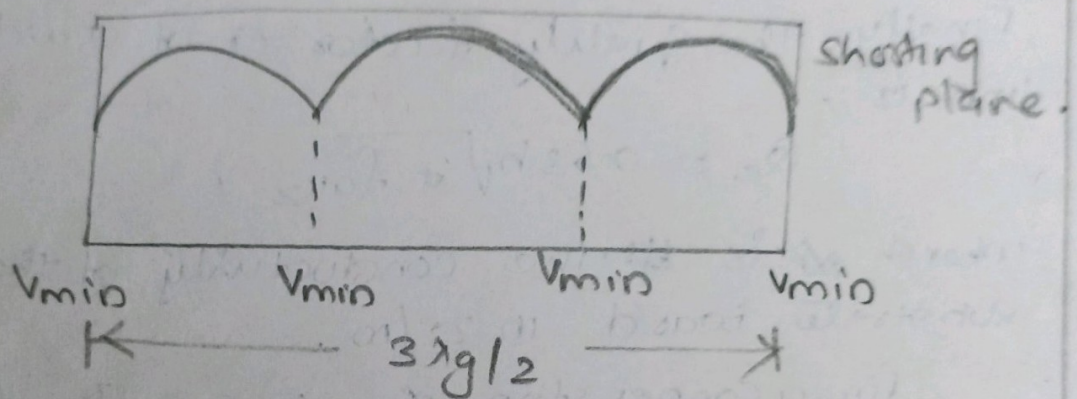
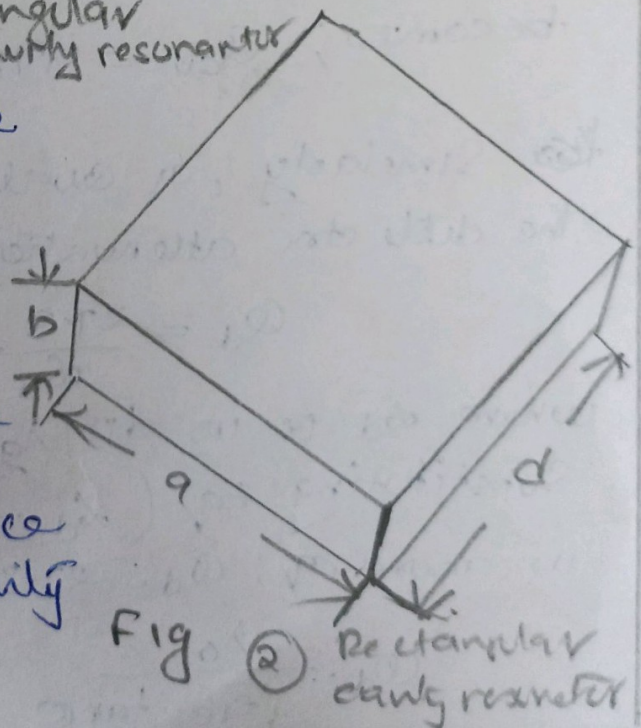


Fig (1) Rectangular cavity resonator

When another shorting plate is kept at a distance of a multiple of $\lambda/2$ then the hollow space so formed can support a signal which bounces back and forth between the two shorting plates. This results in resonance and hence the hollow space is called cavity and the resonator as the cavity resonator.



In practice, the rectangular-cavity resonator, circular cavity resonator and reentrant-cavity resonator are commonly used in many microwave applications. A given resonator has an infinite number of resonant modes, and each mode corresponds to a definite resonant frequency. When the frequency of an impressed signal is equal to a resonant frequency, a maximum amplitude of the standing wave occurs, and the peak energies stored in the electric & magnetic fields are equal.

The mode having the lowest resonant frequency is known as the dominant mode.

RECTANGULAR CAVITY RESONATOR.

The electromagnetic field inside the cavity should satisfy Maxwell's equations, subject to the boundary conditions that the electric field is tangential to the walls and the magnetic field normal to the metal walls must vanish. The geometry of a rectangular cavity is shown below in fig 1.

The wave equations in the rectangular resonator should satisfy the boundary condition of the zero tangential E at four of the walls. It is not necessary to choose the harmonic functions in z to

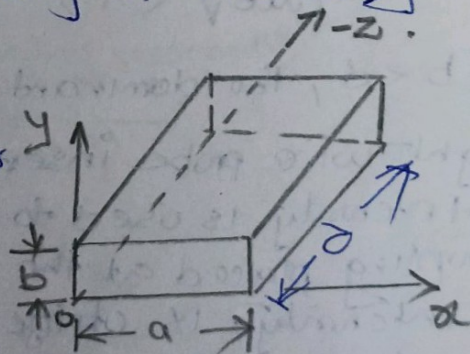


Fig. Co-ordinates of a rectangular cavity.

satisfy this condition at the remaining two ~~and~~ end walls. These functions can be found if,

$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right) \quad (TE_{mnp}) \quad \text{--- (1)}$$

where

$m = 0, 1, 2, 3, \dots$ represents the number of the half wave variations periodically in the x direction

$n = 0, 1, 2, 3, \dots$ represents the no. of half wave periodically in the y direction.

$P = 0, 1, 2, 3, 4, \dots$ represents the number of the half wave periodically in the z direction.

and,

$$E_z = E_{0z} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{P\pi z}{d}\right) \quad (TM_{mnp}) \quad (2)$$

where $m = 1, 2, 3, 4, \dots$

$n = 1, 2, 3, 4, \dots$

$P = 0, 1, 2, 3, \dots$

The separation equation for both TE and TM modes

$$\text{is given by } k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{P\pi}{d}\right)^2 \quad (3)$$

for a lossless dielectric, $k^2 = \omega^2 \mu \epsilon$; therefore, the resonant frequency is expressed by,

$$f_r = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{P}{d}\right)^2} \quad (TE_{mnp}, TM_{mnp}) \quad (4)$$

for $a > b < d$, the dominant mode is the TE_{101} mode.

→ A straight wire probe inserted at the position of max. electric intensity is used to excite a desired mode, and the loop coupling placed at the position of maximum magnetic intensity, is utilized to launch a specific mode.

Fig. below shows the methods of excitation for the rectangular resonator. The maximum amplitude of the standing wave occurs when the frequency of the impressed signal is equal to the resonant frequency.

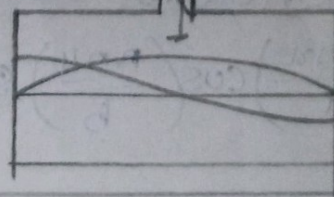
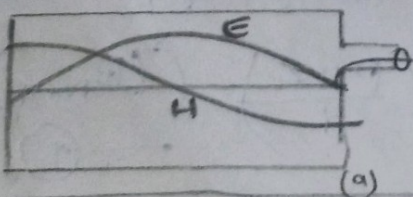


Fig. Methods of exciting wave modes in a resonator.

Q FACTOR OF A CAVITY RESONATOR

The Quality Factor Q is a measure of the frequency selectivity of a resonant or antiresonant circuit and is defined as,

$$Q = 2\pi \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}} = \frac{\omega W}{P} \quad \text{--- (1)}$$

where W is the maximum energy stored
 P is the average power loss.

At resonant frequency, the electric and magnetic energies are equal and in time quadrature, when the electric energy is maximum, the magnetic energy is zero and vice versa. The total energy stored in the resonator is obtained by integrating the energy density over the volume of the resonator.

$$W_e = \int_V \frac{\epsilon}{2} |E|^2 dV = W_m = \int_V \frac{\mu}{2} |H|^2 dV = W \quad \text{--- (2)}$$

where $|E|$ and $|H|$ are the peak values of the field intensities.

The average power loss in the resonator can be evaluated by integrating the power density as given in eq (as $P = \frac{1}{2} |H_t|^2 R_s$ -- this is avg power density for a good conductor) over the inner surface of the resonator.

$$\text{Hence, } P = \frac{R_s}{2} \int_S |H_t|^2 dA \quad \text{--- (3)}$$

where H_t is the peak value of the tangential magnetic component intensity and R_s is the surface resistance of the resonator.

substitution of eq (2) and eq (3) in eq (1), gives

$$Q = \frac{\omega \mu \int_V |H|^2 dV}{R_s \int_V |H_t|^2 dV} \quad \text{--- (4)}$$

Since the peak value of the magnetic intensity is related to its tangential and normal components by,

$$|H|^2 = |H_t|^2 + |H_n|^2$$

where H_n is the peak value of the normal magnetic intensity, the value of $|H_t|^2$ at the resonator walls is approximately twice the value of $|H|^2$ averaged over the volume. So the Q of a cavity resonator as shown in eq (4) can be expressed approximately by,

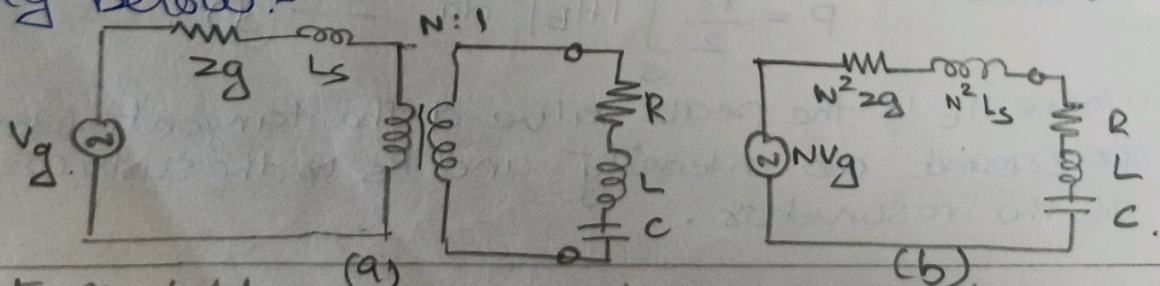
$$Q = \frac{\omega \mu (\text{volume})}{2 R_s (\text{surface area})} \quad \text{--- (5)}$$

An unloaded resonator can be represented by either a series or a parallel resonant circuit. The resonant frequency and the unloaded Q_0 of a cavity resonator are,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{--- (6)}$$

$$Q_0 = \frac{\omega_0 L}{R} \quad \text{--- (7)}$$

If the cavity is coupled by means of an ideal $N:1$ transformer and a series inductance L_s to a generator having internal impedance Z_g , then the coupling circuit and its equivalent are as shown in Fig below:-



cavity coupled to generator (a) coupling circuit (b) ext circuit.

The loaded Q_L of the system is given by,

$$Q_L = \frac{\omega_0 L}{R + N^2 Z_g} \quad \text{for } |N^2 Z_g| \ll |R + N^2 Z_g| \quad \text{--- (8)}$$

The coupling coefficient of the system is defined as,

$$K = \frac{N^2 Z_g}{R} \quad \text{--- (9)}$$

and the loaded Q_L would become

$$Q_L = \frac{\omega_0 L}{R(1+K)} = \frac{Q_0}{1+K} \quad \text{--- (10)}$$

rearranging of eq (10), yields

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} \quad \text{--- (11)}$$

where $Q_{ext} = Q_0/K = \omega_0 L/(KR)$ is the external Q .

There are 3 types of coupling coefficients:

1. Critical coupling: If the resonator is matched to the generator, then,

$$K = 1 \quad \text{--- (12)}$$

The loaded Q_L is given by,

$$Q_L = \frac{1}{2} Q_{ext} = \frac{1}{2} Q_0 \quad \text{--- (13)}$$

2. Overcoupling: If $K > 1$, the cavity terminals are at a voltage maximum in the input lines at resonance. The normalized impedance at the voltage maximum is the standing-wave ratio ρ . That is,

$$K = \rho \quad \text{--- (14)}$$

The loaded Q_L is given by,

$$Q_L = \frac{Q_0}{1+\rho} \quad \text{--- (15)}$$

3. Undercoupling: If $K < 1$, the cavity terminals

are at a voltage minimum and the input terminal impedance is equal to the reciprocal of the standing wave ratio. That is,

$$K = \frac{1}{\rho} \quad (16)$$

The loaded Q_L is given by,

$$Q_L = \frac{\rho}{\rho + 1} Q_0 \quad (17)$$

The relationship of the coupling coefficient K and the standing-wave ratio is shown in Fig below:

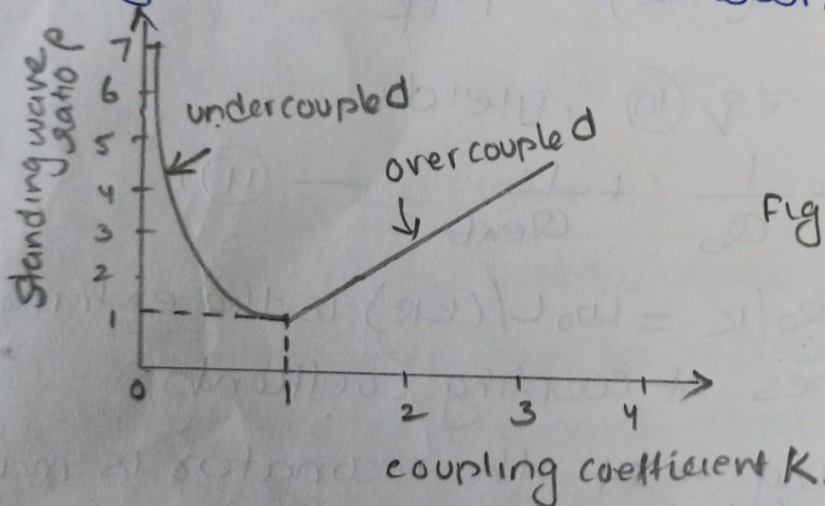


Fig: Coupling coefficient versus SWR.

UNIT - 3 II

WAVE GUIDE COMPONENTS AND APPLICATIONS - I

INTRODUCTION

A microwave circuit ordinarily consists of several microwave components connected in some way to achieve the desired Tx of a microwave signal. The interconnection of two or more microwave devices may be regarded as a microwave junction.

Interconnection of two or more microwave components is called a microwave junction.

Microwave systems normally consist of several microwave components including the source and the load being connected to each other by wave guide or co-axial transmission line system.

The rectangular and circular wave guides, cavity resonators etc. that are also the microwave components (passive). Apart from these, there are other components like, joints, wave guide junctions, posts, screws, diodes, directional couplers, ferrite devices, phase shifters, filters etc.

Commonly used microwave junctions include such as wave guide TEES, directional couplers.

COUPLING MECHANISMS :-

A waveguide operates differently from an ordinary transmission line. Therefore special devices must be used to put energy into a waveguide at one end and remove it from other end.

Coupling means to couple a co-axial line to a waveguide, to couple the signal to a waveguide, or to couple waveguides to each other by means of slots in common walls.

The three devices used to inject or remove energy from waveguide are : probes

Loops

slots are also called

← Slots (Aperture)

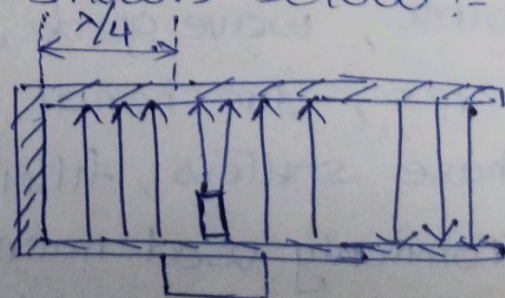
Apertures or windows.

D. PROBE COUPLING :-

When a small probe is inserted into a waveguide and supplied with microwave energy, it acts as a quarter wave antenna. Current flows in the probe and sets up an E field such as the one shown below :-



(a)



(b)

Fig: Probe coupling in a rectangular waveguide.

The E-lines detach themselves from the probe when the probe is located at the point of highest efficiency, the lines set up an E-field of considerable intensity.

The correct position of placing the probe is placing it at a distance of $\lambda/4$ from the shorted end of the waveguide and the centre of broader dimension of the waveguide because at that point electric field is max. This probe will now act as an antenna which is polarized in the plane parallel to that of electric field.

In many applications a lesser degree of energy transfer, called loose coupling, is desirable. The amount of energy transfer can be reduced by decreasing the length of the probe, by moving it out of the centre of the E-field or by shielding it.

The size and shape of the probe determines its frequency, bandwidth and power handling capability. As the diameter of probe increases the bandwidth increases. The greater power handling capability is directly related to the increased surface area.

Removal of energy from a waveguide is a reversal of the injection process using the same probe.

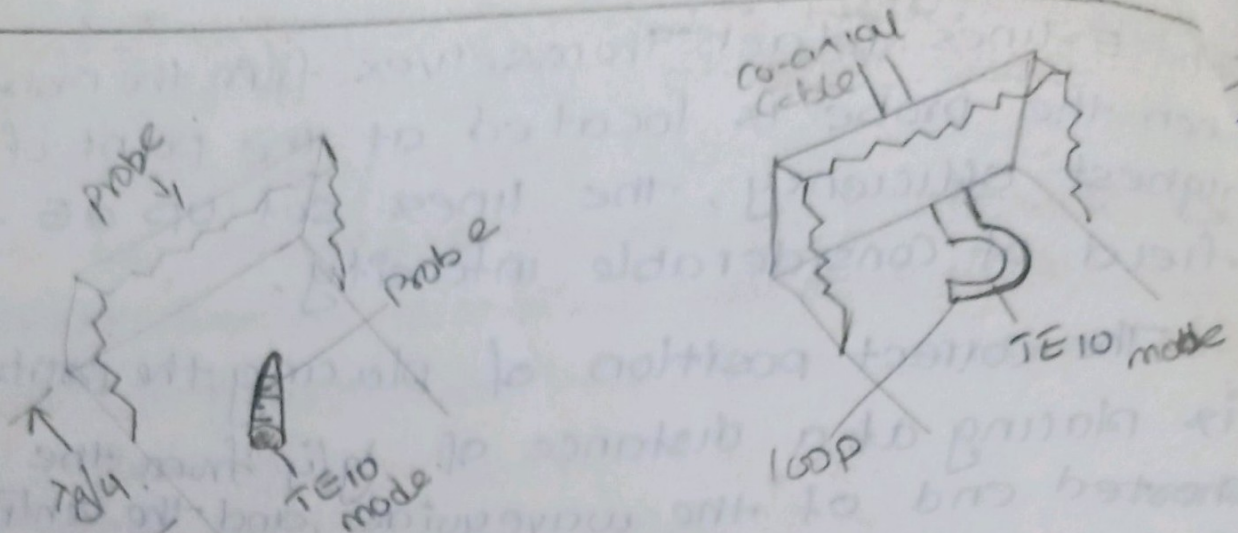


fig: coupling probe and loop.

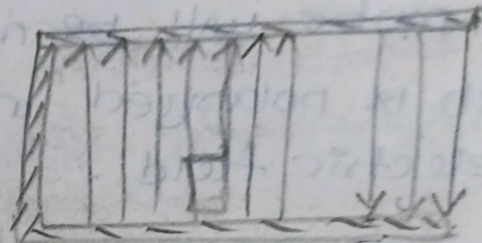


fig: probe coupling in a rectangular waveguide.

LOOP COUPLING:-

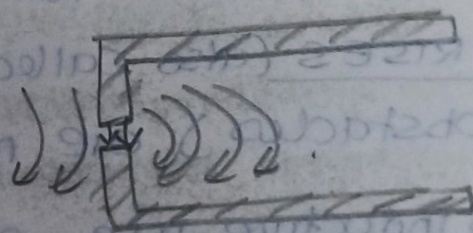
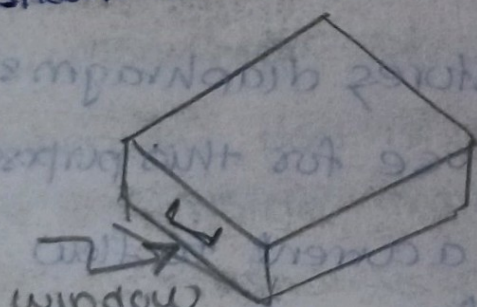
Another way of injecting energy into a waveguide is by setting up an H-field in the waveguide. This can be accomplished by inserting a small loop which carries a high current into the waveguide. A mag field builds up around the loop and expands to fit the waveguide. If the frequency of the current in the loop is within the bandwidth of the waveguide, energy will be transferred to the w.g.

The coupling loop placed at the centre of shorted end plate of the waveguide can also be used to launch TE_{10} mode, i.e., coupling is achieved by means of a loop antenna located in a plane perpendicular to the plane of the probe.

It is thus seen that probes couple primarily to the electric field and loops to a mag. field but in each case both are set up because electric and magnetic fields are inseparable.

APERTURE OR SLOT COUPLING :-

Slots or apertures are used when very loose (insufficient) coupling is desired as shown below.



Aperture by Aperture Coupling in waveguide

In this method, energy is coupled through a small slot, in the waveguide and the E-field expands into the waveguide. minimum reflection occurs when energy is injected or removed if the size of the slot is properly proportional to the frequency of the energy.

WAVE GUIDE DISCONTINUITIES.

i) WAVE GUIDE IRISES:

In any wave guide system, when there is a mismatch there will be reflections. In transmission lines, in order to overcome this mismatch lumped impedances or stops of required value are placed at precalculated points.

→ In waveguides also, some discontinuities are made use of for matching purposes.

Any susceptance appearing across the guide, causing mismatch (and producing of standing waves) needs to be cancelled by introducing another susceptance of the same magnitude but of opposite nature.

IRISES (also called apertures, diaphragms or obstacles) are made use for this purpose.

→ An Inductive iris allows a current to flow where none flowed before. The iris is placed in a position where the magnetic field is strong (or where electric field is relatively weak) because the plane of polarization of electric field is parallel to the plane of iris, the current flow due to iris causes a magnetic field to be set up. Energy storage of mag field takes place and there is an $\uparrow$ in inductance at that point of W.G.

TUNING SCREWS AND POSTS:-

When a metallic cylindrical post is introduced into the broader side of the W.G., it produces the same effect as an iris. i.e., providing lumped reactance at that point.

→ If the post extends only a short distance i.e. $< \lambda_g/4$ into the wave guide, it behaves capacitively (fig 2.a). and this capacitive susceptance increases with depth of penetration.

→ When the depth is equal to $\lambda_g/4$, the post acts as a series resonant circuit. (fig 2.b).

→ If it is $> \lambda_g/4$, the post acts ~~as a~~ inductively (fig 2.c). and this inductive susceptance decreases when the post is moved further away from the centre of the wave guide.

→ When the post is extended completely across the waveguide, the post becomes inductive (fig 2.d).

The susceptance vs penetration curve is shown in fig (2.e).

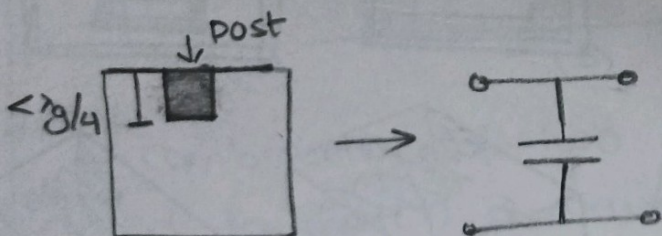
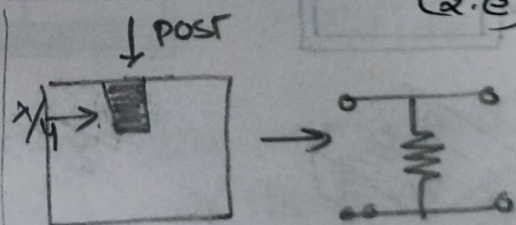


fig (2.a)
(capacitive)



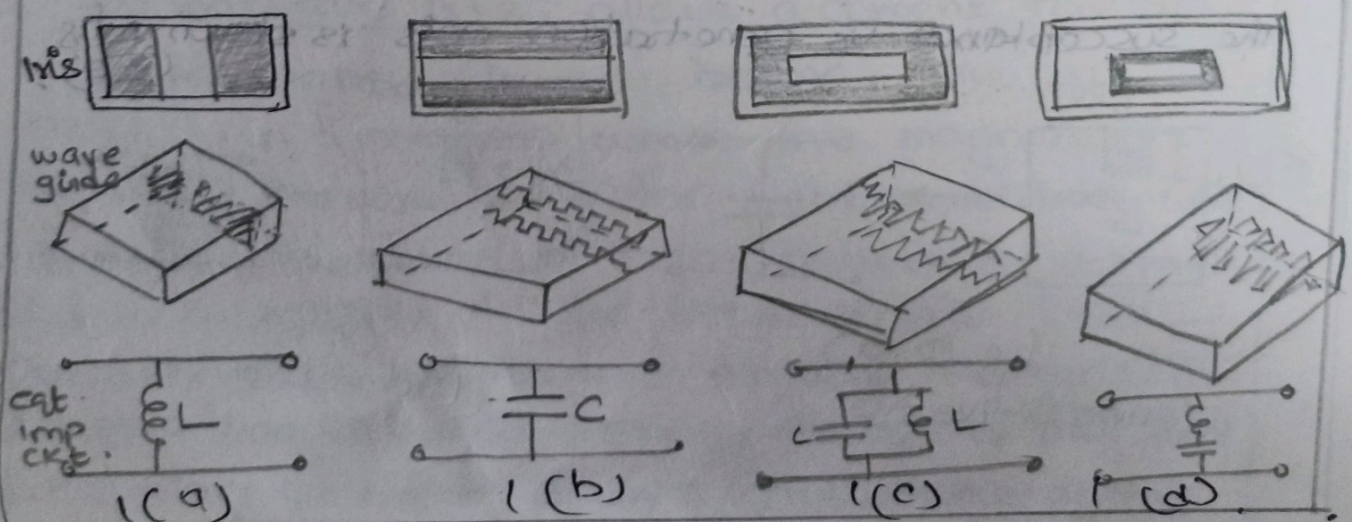
(2.b)
(inductive)
Resonant circuit

Capacitive iris, it is seen that the potential ϕ existed b/w the top and bottom walls of the waveguide now exists between surfaces which are closer and therefore the capacitance has increased at that point. The capacitance iris is placed in a position where the electric field is strong.

The inductive and capacitive iris if combined suitably, the inductive and capacitive reactances introduced will be equal and the iris becomes a parallel resonant circuit (fig c).

For the dominant mode, the iris presents a high impedance and the shunting effect for this mode will be negligible. Other modes are completely attenuated and the resonant iris acts as a BPF to suppress unwanted modes.

Fig(d) shows a series resonant iris which is supported by a non-metallic material and is transparent to the flow of microwave energy.



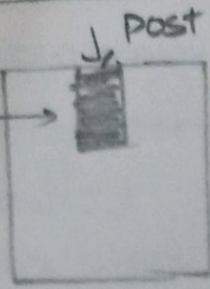


fig (2.c)
(Inductive)

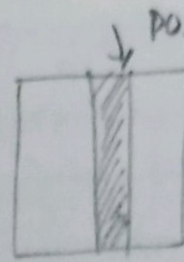
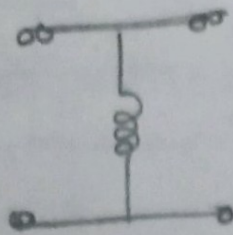
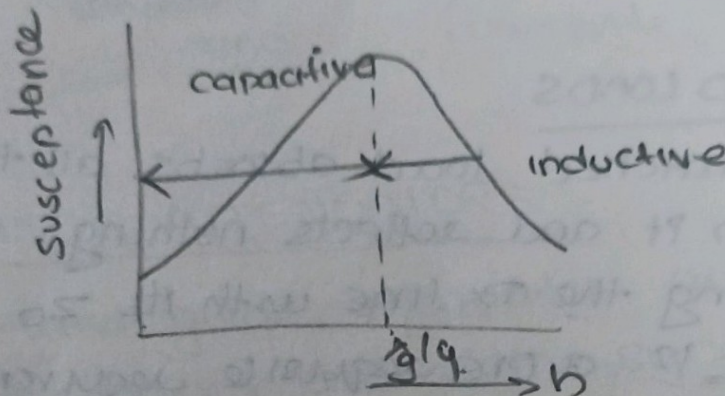
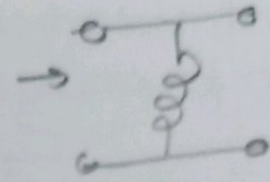
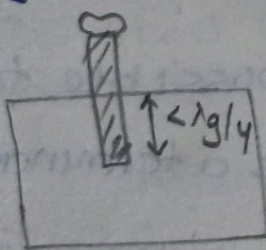


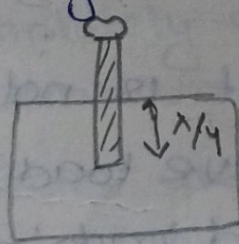
fig (2.d)
inductive



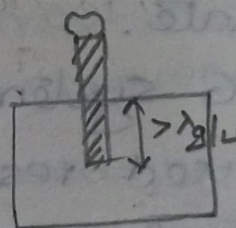
The advantage of the post over an iris is that it is readily adjustable. An adjustable post is known as a Screw or slug. The adjustable or tuning screws are shown in fig below:-



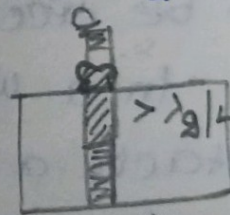
(a)



(b)

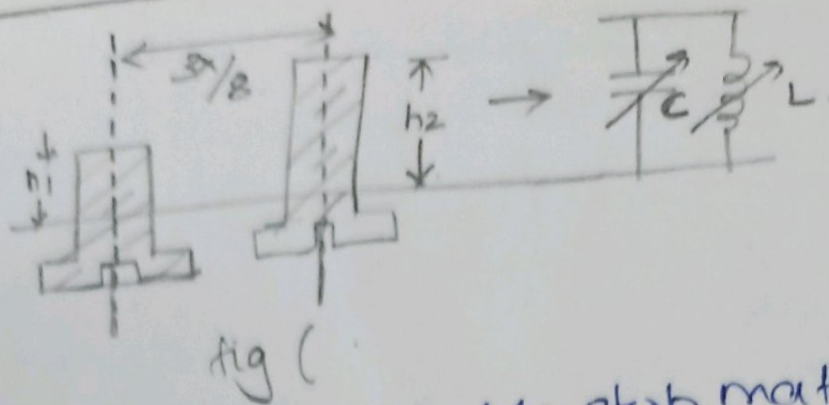


(c)



(d)

A combination of two screws $\lambda/4$ apart can be used to match a wave guide to its load similar to use of two fixed stubs in a transmission line. A very effective wave guide matcher can be realised when two tuning screws are placed in close proximity separated by $3\lambda/8$.



This is similar to double stub matching in Tx lines.

(iii) MATCHED LOADS

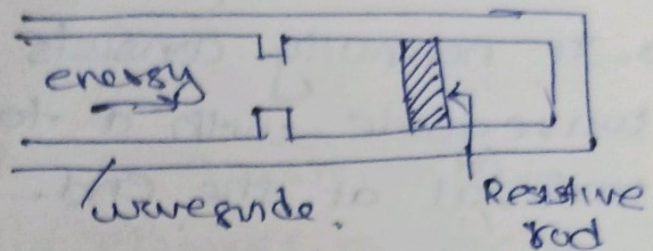
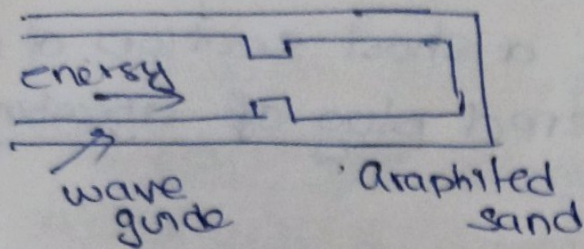
The matched load absorbs all the incident power upon it and reflects nothing. This is eqt. to connecting the Tx line with its Z_0 . Matching of the load is a prerequisite requirement in a waveguide system as mismatching in waveguide will cause reflection of energy. Hence the load should be capable of absorbing the energy without reflection, if measurement are to be accurate.

In a WG system it is not possible to attach a fixed resistive load as a termination.

Methods of matched loads:-

- 1) placing a graphited sand at the end of the waveguide, can dissipate energy to achieve SWRs of less than 1.01. (fig 3(a))
- 2) placing a resistive rod at a point in the w.g. where electric field strength is max. (fig 3(b))

3). placing a wedge of resistive material (powdered iron or carbon mixed with a binder deposited on a dielectric strip) in the form of a taper that can act as a termination.



WAVE GUIDE ATTENUATORS :-

In wave guide, attenuators are used to reduce the power load of the signal so that overloading of a particular stage can be avoided.

Attenuators are used for perfect matching i.e. the microwave power in a waveguide be absorbed completely without any reflection and also insensitive to frequency.

Attenuators are commonly used for measuring power gain or loss in dBs, for providing isolation between instruments, for reducing the power l/p to a particular stage to prevent overloading.

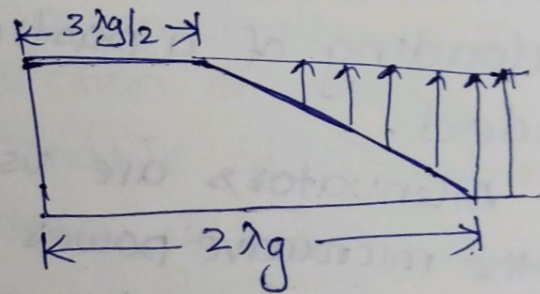
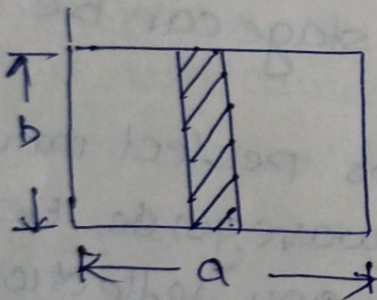
Attenuators are classified as :-

- 1) Fixed attenuators
- 2) Variable "
- 3) Ferrite " .

FIXED ATTENUATORS:- They are used where fixed attenuation is to be provided.

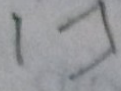
- It is also called matched load or terminator, if it absorbs all the incident power.
- It normally consists of a short section of a waveguide with a tapered plug of absorbing material at the end.
- The tapering is done for providing a gradual transition from the wave guide medium to the absorbing medium thus reducing the reflection occurring at the media interface.

A fixed attenuator is as shown:-



VARIABLE ATTENUATOR:-

- It is used to provide continuous or step wise variable attenuation.
- For Rectangular waveguides, these attenuators are of Flap type or Vane type.
- For circular wave guides rotary type is used.



The Flap type attenuator is as shown above.

It consists of a resistive element or disc inserted into a longitudinal slot cut along the centre of the wider dimension of the guide.

→ The Flap is mounted on the hinged arm allowing it to descent into the centre of the wave guide.

→ The degree of attenuation is determined by the depth of insertion of the Flap.

Disadvantage of the Flap attenuator is that, it is not a precision attenuator, its dial needs to be calibrated against a standard.

The ⁽ⁱⁱ⁾ VANE TYPE Attenuator consists of a glass vane with a coating of aquadag or carbon similar to a fixed vane attenuator.

→ If this vane used at the centre is made movable, it can be used as a variable attenuator.

→ The vane positioned at the centre of the

waveguide can be moved laterally from the center where it provides maximum attenuation to the edges where the attenuation is considerably reduced since the electric field lines are always concentrated at the centre of the W.G.

- The vane is tapered at both ends for matching the attenuator to the waveguide.
- An adequate match is obtained if the taper length is made equal to $\lambda_g/2$.
- The amount of attenuation is frequency sensitive and also has to be calibrated against a precision (std) attenuator.

A movable vane attenuator is shown below:-

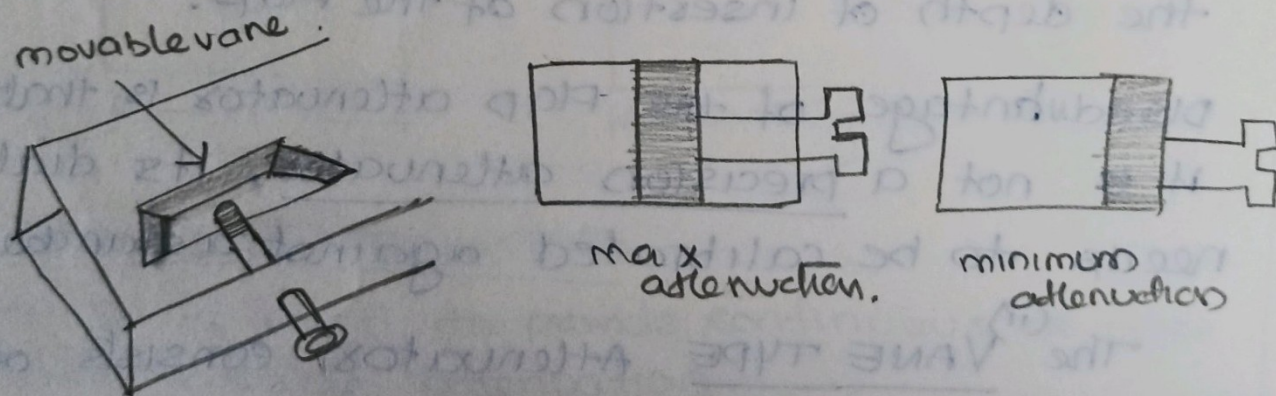


fig:- movable vane attenuator

ROTARY VANE ATTENUATOR.

A resistive rotary vane attenuator provides precision attenuation with an accuracy of $\pm 2.1\%$ of the indicated attenuation over the operating frequency range. It consists of three vanes.

→ The central vane rotating type placed in the central section of a circular waveguide arrangement tapered at both ends.

The other two vanes are in the rectangular sections as shown in fig below.

→ When all the three vanes are aligned their planes are at 90° to the direction of electric field, hence there is no or zero attenuation.

→ Vane 1 prevents any horizontal polarisation and hence electric field at the o/p of the vane 1 is vertically polarised.

→ The centre vane 2 is rotating type and if it is rotated by an angle θ , the $E \sin \theta$ component is attenuated and $E \cos \theta$ component is present, at the o/p

of vane 2, and the final o/p of the attenuator becomes $E \cos^2 \theta$ which has the same polarization as the i/p wave.

→ The attenuation due to this rotary vane attenuator is then equal to $20 \log \cos^2 \theta = 40 \log \cos \theta$ that is independent of freq and is precise.

$$\underline{\hspace{10em}} \times \underline{\hspace{10em}}$$

WAVE GUIDE PHASE SHIFTERS:-

Many applications require phase shifts to be introduced between two given positions in a wave guide system. The phase shift required may be fixed or variable. Since the phase shift constant β is inversely proportional to guide wavelength λ_g , the magnitude of λ_g could be changed to obtain variable amounts of phase shift. Fixed amounts of phase shifts can be obtained by use of capacitive / inductive irises in the waveguide or by inserting dielectric rods across the diameter of a circular waveguide or by reducing wider dimension of a rectangular waveguide.

The physical construction of a phase shifter is same as that of a vane attenuator. It consists of a dielectric slab or vane specially shaped to minimise reflection effects, inserted through longitudinal slot cut along the wider dimension of a waveguide as shown in Fig below:-

This dielectric slab is made of some low loss material with $\epsilon_r > 1$. Higher the dielectric constant of a medium the more slowly ~~the more~~ slower a microwave signal travels through it.

Fig ①

Since most of the microwave signal in a waveguide travels through the centre of the waveguide, movement of a dielectric slab with $\epsilon_r > 1$ towards the centre of the waveguide means that microwave signal moves more slowly the nearer the slab gets to the centre.

The electric field distribution in the broader dimension of the waveguide will be modified by the dielectric vane so that it is distorted from sinusoidal to that shown in fig ①

If the vane is inserted deeper, there is more change in the medium and there is a greater phase shift. The electric field distribution also shows that the dielectric has the same effect of increasing the broader dimension of the waveguide which reduces the wavelength in the waveguide. The amount of phase shift is maximum when the slab is at the centre and minimum when it's adjacent to the wall of the waveguide. If the dielectric vane is placed such that the vane's inside dimension is parallel to the direction of the electric flux lines.

In the above dielectric vane variable phase shifter the phase shift is changed continuously from one value to another ϕ hence they are also termed as analog phase shifters.

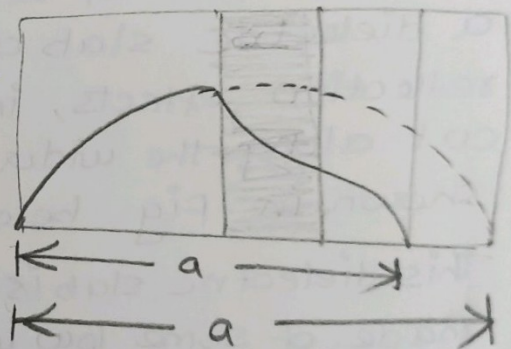


Fig. Electric field distribution.

A precision phase shifter can be realised by a rotary phase shifter useful in microwave measurement. It basically consists of three circular waveguide sections all of which contain one dielectric vane. The centre section is rotatable providing the required phase shift.

It works on the principle of converting a linearly polarised TE_{11} mode into a circularly polarised mode.

WAVEGUIDE COMPONENTS AND APPLICATIONS.

'Ferrite' is a generic name given to ferromagnetic (or ferrimagnetic) insulators with specific resistivities as high as 10^{14} times that of metals and with dielectric constants around 10 to 15 times greater and relative permeabilities of several thousand.

4.1 FERRITES - Composition and characteristics.

Ferrites are non metallic materials, which are made by sintering a mixture of metallic oxide and have the general chemical composition $M-O-Fe_2O_3$, where M is a divalent metal such as Zn, Ni, Co, Mn, etc or a mixture of these. They are obtained by firing powdered oxides of materials at 1100°C or more and pressing them into different shapes. This processing gives them the added characteristics of ceramic insulators so that they can be used at microwave frequencies.

Ferrites have atoms with large number of spinning electrons resulting in strong magnetic properties. These magnetic properties are due to the magnetic dipole moment associated with the electron spin. Because of the above properties, ferrites find application in a number of microwave devices to reduce reflected power, for modulation purposes and in switching circuits. Because of high resistivity they can be used up to 100 GHz .

Ferrites have one more peculiar property which is useful at microwave frequencies i.e., the non reciprocal property when two circularly polarized waves one rotating clockwise and other anticlockwise are made to propagate through ferrite, the material reacts differently to the two rotating fields, thereby presenting different effective permeabilities to both the waves i.e., ϵ_1, μ_1 for left circularly polarized wave and ϵ_2, μ_2 for right circularly polarized wave.

4.2 FARADAY ROTATION

Ferrite is a family of $MeO \cdot Fe_2O_3$, where Me is a divalent iron metal. When a piece of ferrite is affected by a dc magnetic field, the ferrite exhibits Faraday rotation. It does so because the ferrite is a nonlinear material and its permeability is an asymmetric tensor, as expressed as, $B = \hat{\mu} H$.

Consider an infinite lossless medium. A static magnetic field B_0 is applied along the z direction. When a dc magnetic field is applied to a ferrite, the unpaired electrons in the ferrite material tend to line up with the dc field because of their magnetic dipole moment. However, the nonreciprocal precession of unpaired electrons in the ferrite causes their relative permeabilities, (μ_r^+, μ_r^-) to be unequal and the wave in the ferrite is then circularly polarized.

If μ_r^+ is much larger than μ_r^- ($\mu_r^+ \gg \mu_r^-$) the wave in the ferrite is rotated in the clockwise direction. Consequently, the propagation phase constant β^+ for the forward direction differs from the propagation constant β^- for the backward direction. By choosing the length of the ferrite slab and the dc magnetic field so that,

$\omega = (\beta^+ - \beta^-)l = \pi/2$, a differential phase shift of 90° for the two directions of propagation can be obtained.

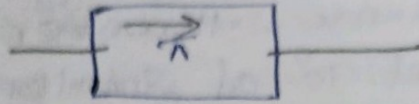
4.3 FERRITE COMPONENTS :-

Microwave circulators, isolators and gyrators are nonreciprocal transmission devices that use the property of Faraday rotation in the ferrite material.

3.1. GYRATOR :-

An Important canonical nonreciprocal component is the Gyrator, which is a two-port device having a 180° differential phase shift. The schematic symbol for a gyrator is shown in Fig 4.1, and the scattering matrix for an ideal gyrator is

$$S = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$



which shows that it is lossless, matched, and nonreciprocal.

Fig 4.1 - Symbol for gyrator which has differential phase shift of 180° .

A Gyrator is defined as a two port device that has a relative difference in phase shift of 180° for transmission from port 2 to port 1. A gyrator may be obtained by employing the non-reciprocal property of Faraday rotation.

Construction:- As shown in Fig 4.2, A gyrator consists of a rectangular waveguide with a 90° twist connected to a circular guide, which in turn is connected to another rectangular guide at the other end. The two rectangular guides have the same orientation at the input ports. The circular guide contains a thin cylindrical rod of ferrite with the ends tapered to reduce reflections. A static axial magnetic field is applied so as to produce 90° Faraday rotation of the TE_{11} dominant mode in the circular guide.

Operation:-

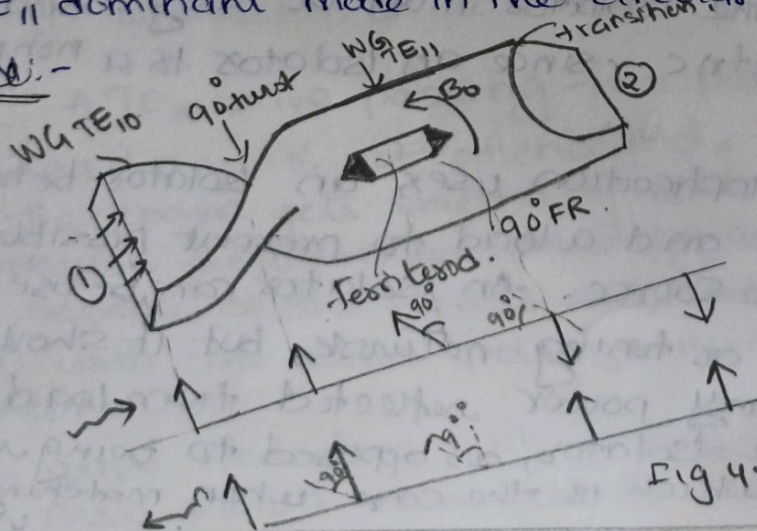


Fig 4.2 - microwave gyrator.

OPERATION:- Consider a wave propagating from left to right. In passing through the twist the plane of polarization is rotated by 90° in counter clockwise direction. If the ferrite produces an additional 90° of rotation, the total angle of rotation will be 180° . For a wave propagating from right to left, the Faraday rotation is still 90° in the same sense. However in passing through the twist, the next 90° of rotation is in a direction to cancel the Faraday rotation. Thus the transmission from post 2 to post 1, there is no net rotation of the plane of polarization. The 180° rotation for ~~transmission~~ transmission from post 1 to post 2 is equivalent to an additional 180° of phase shift. Since it reverses the polarization of the field, it is apparent, then that the device just described satisfies the definition of gyrator.

4.3.2 MICROWAVE ISOLATOR.

One of the most useful microwave ferrite components is the Isolator, which is a 2 port device having unidirectional transmission characteristics. The S matrix for an ideal Isolator has the form,

$$[S] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

indicating that both ports are matched, but transmission occurs only in the direction from port 1 to port 2. Since $[S]$ is not unitary, the Isolator must be lossy. And of course $[S]$ is not symmetric, since an Isolator is a nonreciprocal component.

A common application uses an Isolator between a high power source and a load to prevent possible reflections from damaging the source. An Isolator can be used in place of a matching or tuning network, but it should be realized that any power reflected from load will be absorbed by the Isolator, as opposed to being reflected back to the load, which is the case when matching n/w is used.

Construction The construction of Isolator (Fig 4.3) is similar to gyrator except that an Isolator makes use of 45° twisted rectangular waveguide instead of 90° twist and 45° ferrite rod instead of 90° in gyrator, a resistive card is placed along the larger dimension of the rectangular waveguide, so as to absorb a wave whose plane of polarization is parallel to the plane of resistive card. The resistive card does not absorb any wave whose plane of polarization is perpendicular to its own plane.

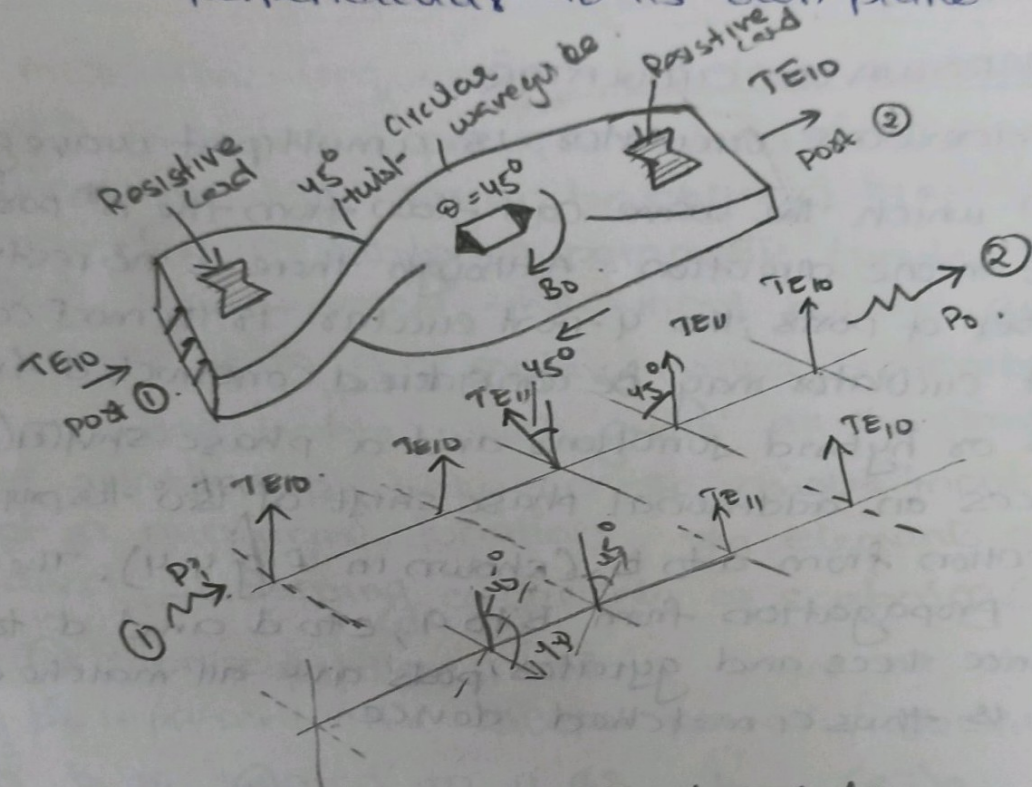


Fig. construction details of Isolator.

Operation. A TE_{10} wave passing from post ① through the resistive card and is not attenuated. After coming out of the card, the wave gets shifted by 45° because of the twist in anticlockwise direction and then by another 45° in clockwise direction because of the ferrite rod, and hence comes out at post ② with the same polarization as at post ① with out any attenuation.

A TE_{10} wave fed from post ② gets a pass from the resistive card placed near post ② since the plane of

polarization of the wave is perpendicular to the plane resistive card. Then the wave gets rotated by 45° due to Faraday rotation in clockwise direction and further gets rotated by 45° in clockwise direction due to the twist in the waveguide. Now the plane of polarization of the wave will be parallel with that of the resistive card and hence the wave will be completely absorbed by the resistive card and the output at port ① will be zero. This power is dissipated in the card as heat.

4.3.3. MICROWAVE CIRCULATOR

A microwave circulator is a multipost waveguide junction in which the wave can flow from the n^{th} post to the $(n+1)^{\text{th}}$ post in one direction. Although there is no restriction on the number of posts, the 4-post circulator is the most common. A four post circulator may be considered constructed from two magic tees or hybrid junctions and a phase shifter (gyrator) which produces an additional phase shift of 180° for propagation in the direction from a to b. (shown in Fig 4.4). The electrical lengths of propagation from b to c, c to d and d to a are equal. Since tees and gyrator ports are all matched, the circulator is thus a matched device.

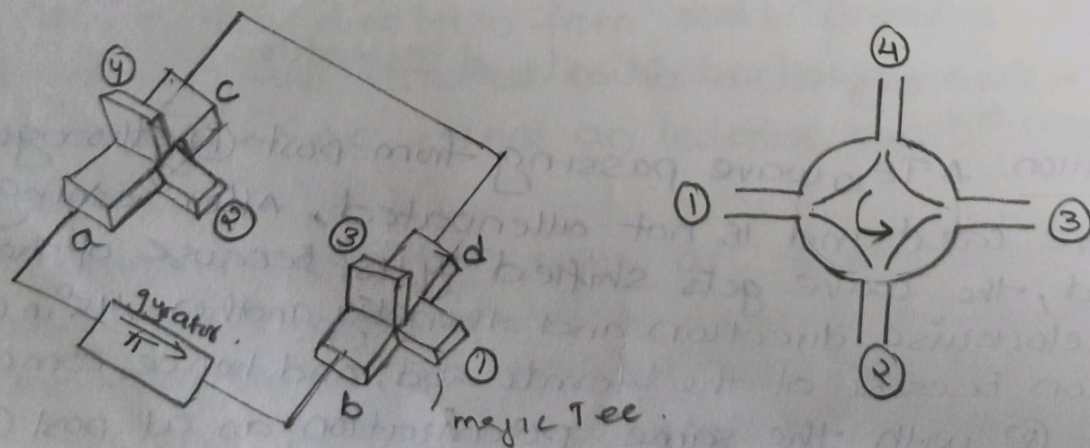


Fig 4.4. A 4 post circulator using magic tee.

Operation :- Consider a wave incident on port 1, it splits into 2 equal in phase waves propagating inside arms, each at a and c in phase thereby emerging in the 'adder' arm of another Tee i.e. port 2. The incident wave in port 2 will split in two waves which arrive at b and d with 180° phase shift and as a result, shall be added in subtractor arm of the second magic Tee i.e. port 3 and so on.

4.4 - SCATTERING MATRIX - SIGNIFICANCE & FORMULATION

At microwave frequencies, we talk about travelling waves with associated powers instead of voltages & currents and the microwave junction can be defined by what are called as S-parameters or Scattering parameters (similar to H, Y, Z parameters). We represent the input powers and output powers in the form of a matrix called Scattering matrix. It is a square matrix which gives all the combinations of power relationships between the various input and output ports of a microwave junction. The elements of this matrix are called scattering coefficients or scattering (S) parameters.

Let a_1 represent the incident wave at the junction and b_1 represent the output power (reflected power at 1st port) then b_1 is related to a_1 as, $b_1 = [S_{11}] a_1$.

where S_{11} represent the reflection coefficient of 1st line.

1 = reflection from 1st line and

i = Source connected at ith line.

the contribution to the outward travelling wave in

UNIT- VIII

MICROWAVE TUBES.

5.0

INTRODUCTION:-

Conventional triodes, and pentodes are useful only at low microwave frequencies. Special tubes would be required even at UHF frequencies (300-3000 MHz), as conventional tubes have certain limitations at microwave frequencies. All tubes including UHF tubes are bound to fail at higher microwave frequencies due to their limitations at those frequencies.

At microwave frequencies, the size of electronic device required for generation of microwave energy becomes smaller and smaller. This results in lesser power handling capability and increased noise levels. Electronic devices such as tubes and transistors will be required even at microwave frequencies. Tubes can provide higher o/p powers while transistors are smaller have lesser noise, better reliability with reduced output power levels.

In this chapter microwave tubes that have enjoyed the greatest technological success in microwave generation and amplification will be discussed. These are the cavity klystron, Reflex klystron, Travelling wave tube and the magnetron.

5.1 LIMITATIONS AND LOSSES OF CONVENTIONAL TUBES

With increasing frequency ($> 1 \text{ GHz}$) the conventional tubes (triodes, tetrodes, pentodes) are less useful in amplification and generation because of ~~few~~^{few} important limitations

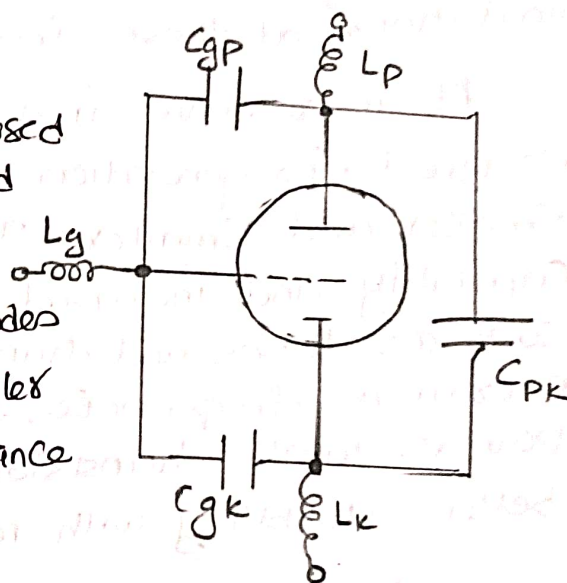
- 1) Inter electrode capacitance effect
- 2) Lead inductance effect
- 3) Transit time effect
- 4) Gain Bandwidth Limitation.

5. Effect due to RF losses.
6. Effect due to radiation losses.

5.1.1. Inter Electrode Capacitance (IEC) Effect.

As frequency increases, the reactance $X_C = \frac{1}{2\pi f C}$ decreases and the output voltage decreases due to shunting effect. Because at higher frequencies X_C becomes almost a short. C_{gp} , C_{gk} and C_{pk} are the IEC's which come into effect and are shown in Fig 5.1.

The effect of IEC can be minimised by reducing the IEC's C_{gk} , C_{pk} and C_{gp} . These can be reduced by decreasing the area of the electrodes (since $C = \frac{\epsilon_0 \epsilon_r A}{d}$) i.e. by using smaller electrodes or by increasing the distance between electrodes.



5.1.2 LEAD INDUCTANCE (LI) EFFECT

Fig 5.1.

As frequencies increases, the reactance $X_L = 2\pi f L$, increases and hence the voltages appearing at the active electrodes are less than the voltages at the base pins. This results in reduced gain for the tube amplifier. L_k , L_p and L_g are the lead inductances that limit the performance of the tube and are shown in Fig 5.1. The effect of LI can be minimised by decreasing L . Since L is proportional to reactance ($X_L = \frac{1}{\omega C}$) L can be decreased by using larger sized short leads with at base pins i.e. by increasing 'A' and decreasing 'l'. This however reduces the power handling capacity.

5.2.3. TRANSIT TIME EFFECT

Another limitation in the application of conventional tubes at microwave frequencies is the electron transit angle between electrodes. The electron transit angle is defined as,

$$\theta_g = \omega \tau_g = \frac{\omega d}{v_0} \quad \text{--- (1)}$$

where $\tau_g = d/v_0$ is the transit time across the gap
 d = separation between cathode and grid.

$v_0 = 0.593 \times 10^6 \sqrt{V_0}$ is the velocity of the electron.

V_0 = d.c voltage.

When frequencies are below microwave range, the transit angle is negligible. At microwave frequencies, however the transit time (or angle) is large compared to the period of the microwave signal, and the potential between the cathode and the grid may alternate from 10 to 100 times during the electron transit.

5.2.4. GAIN BANDWIDTH PRODUCT.

In ordinary vacuum tubes, the maximum gain is generally achieved by resonating the output circuit as shown in Fig 5.2. In the equivalent circuit (Fig 5.2) it is assumed that $r_p \gg \omega L_c$. The load voltage is given by

$$V_e = \frac{g_m V_g}{A + j[\omega C - 1/\omega L_c]} \quad \text{--- (1)}$$

where $A = 1/r_p + 1/R$,

r_p = plate resistance

R = load resistance

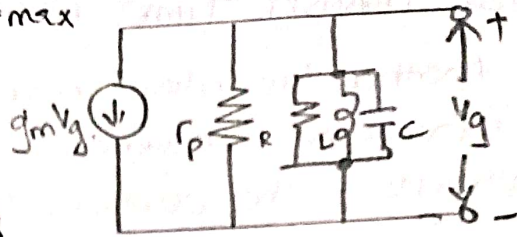
L_c = tuning elements.

The resonant frequency is expressed by,

$$f_r = \frac{1}{2\pi \sqrt{LC}} \quad \text{--- (2)}$$

and the maximum voltage gain A_{max} at resonance by,

$$A_{max} = \frac{g_m}{G} \quad \text{--- (3)}$$



Since the bandwidth is measured at the half power point, the denominator of eq (1) must be related by,

$$G = \omega C + \frac{1}{\omega L}$$

The roots of the quadratic equation are given by,

$$\omega_1 = \frac{G}{2C} - \sqrt{\left(\frac{G}{2C}\right)^2 + \frac{1}{LC}} \quad \text{--- (4)}$$

$$\omega_2 = \frac{G}{2C} + \sqrt{\left(\frac{G}{2C}\right)^2 + \frac{1}{LC}}$$

Therefore the bandwidth can be expressed by

$$BW = \omega_2 - \omega_1 = \frac{G}{C} \quad \text{for } \left(\frac{G}{2C}\right)^2 \gg \frac{1}{LC} \quad \text{--- (5)}$$

Hence the gain bandwidth of the circuit of Fig 5.2 is

$$A_m(BW) = \frac{g_m}{C} \quad \text{--- (6)}$$

It is important to note the gain-bandwidth product is independent of frequency. For a given tube, a higher gain can be achieved only at the expense of a narrower bandwidth.

5.2.5 EFFECT DUE TO RF LOSSES

(a) Skin effect losses. As the frequency is increased, the current flows more and more on the surface of the conductor because of the skin effect. This results in ohmic losses ($I^2 R$) in lead wires as the effective resistance R increases. This can be minimised by the use of conductors having low resistivity and large surface areas: planar electrodes.

Q. (b) Dielectric losses.

This occurs in various types of insulating materials used in the device i.e., spacers, glass envelope, silicon or plastic encapsulation etc. The loss in any of these material is in general given by,

$$P = \pi f \cdot V^2 \epsilon_r \tan \delta \quad \text{--- (1)}$$

where, ϵ_r = relative permittivity of the dielectric
 δ = loss angle of the dielectric.

As f increases, the power loss increases. The remedy for this is to eliminate the tube base to reduce the surface area of glass.

Q. (c) Radiation losses.

Whenever the dimensions of the wire approach the wavelength ($\lambda = c/f$), it will emit radiation that is, radiation losses increase with increase in frequency.

5.3.4. CLASSIFICATION OF MICROWAVE TUBES :-

O TYPE AND M-TYPE. part A

Microwave tubes can be broadly classified into two categories namely, O-type linear tubes and M-tubes. The mechanism of operation in both the cases is essentially the same. The main differences in the two types of tubes are listed below:-

S.no	O type tubes	M-type tubes.
1.	The dc magnetic field is in parallel with the dc electric field, that is used to focus the electron beam.	The dc magnetic field and dc electric field are perpendicular to each other.
2.	These are linear beam tubes.	These are crossed field tubes.

3. O-type tubes derive their name from French TPO (tubes à propagation des ondes) or from the word original.

3. V-type tubes derive their name from after the French TPOM (tubes à propagation des ondes à champs magnétique).

4. DC electric field plays a direct role in the RF interaction process.

4. DC magnetic field plays a direct role in the RF interaction process.

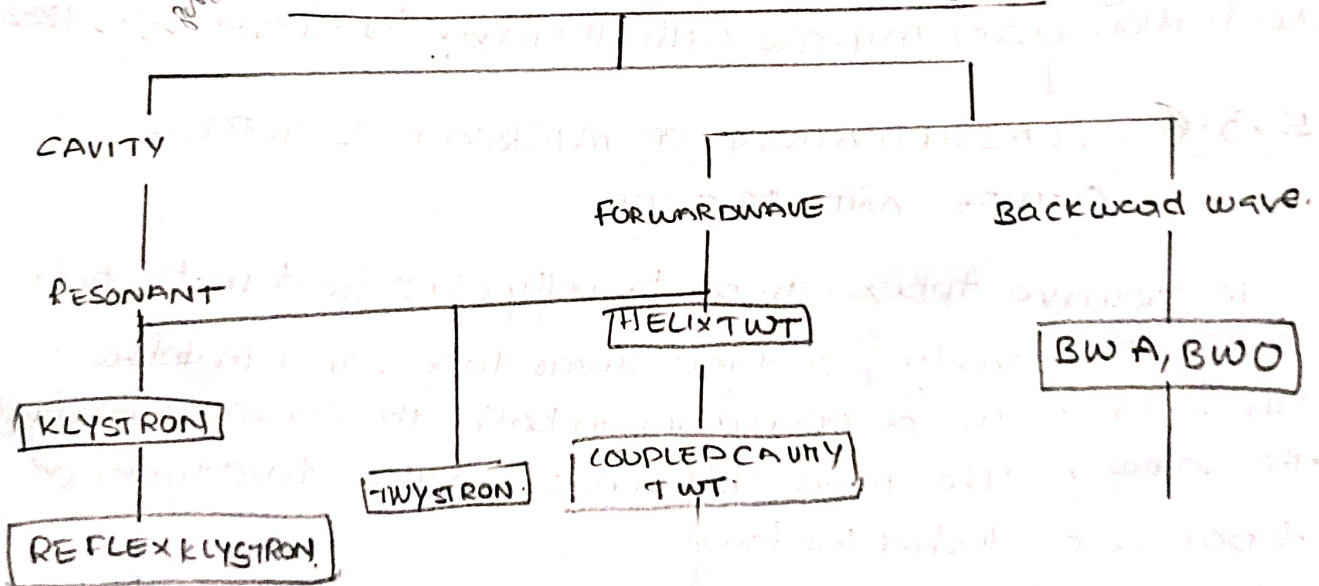
5. Kinetic energy of electrons is converted to RF energy.

5. Potential energy is converted to RF energy.

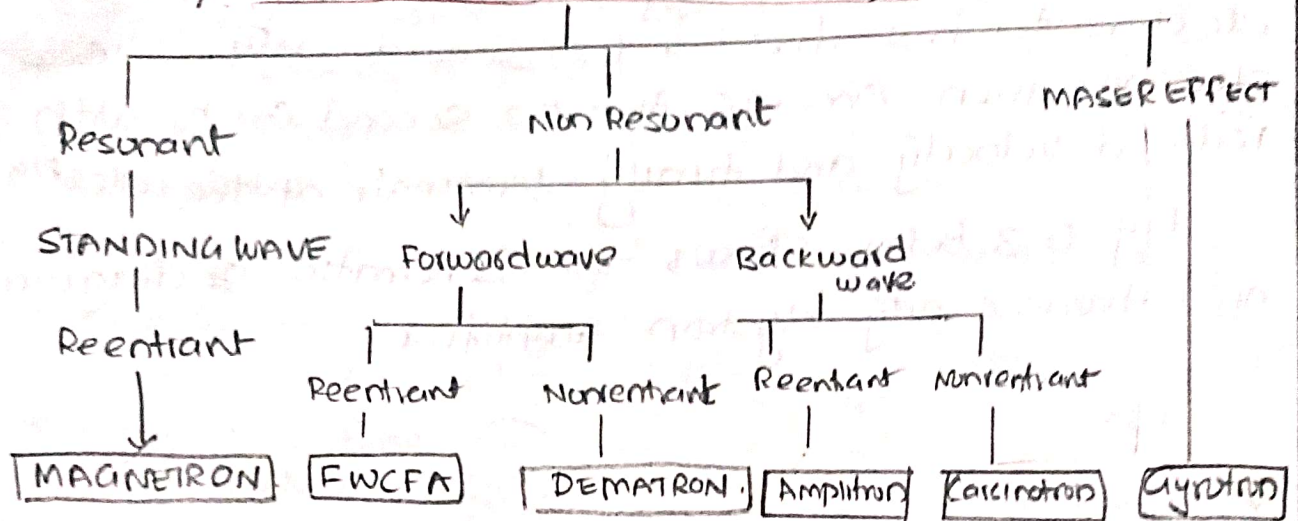
6. Eg. of Linear tubes are 2-cavity klystron, Reflex klystron.

6. Eg of crossed field tubes are ~~TWT~~, magnetron etc.

Linear beam tube (O types)



CROSSED FIELD TUBES. (M-TYPE)



5.4. 2-CAVITY KLYSTRON.

5.4.1 STRUCTURE & operation.

The two cavity klystron is a widely used microwave amplifier operated by the principles of velocity and current modulation. All electrons injected from the cathode arrive at the first cavity with uniform velocity. Those electrons passing the first cavity gap at zeros of the gap voltage pass through with unchanged velocity; those passing through the positive half cycles of the gap voltage undergo an increase in velocity; those passing through the negative swings of the gap voltage undergo a decrease in velocity. As a result of these actions, the electrons gradually bunch together as they travel down the drift space.

The variation in electron velocity in the drift space is known as velocity modulation. The ~~max~~ density of the electrons in the second cavity gap varies cyclically with time. The electron beam contains an ac component and is said to be current modulated. The maximum bunching should occur approximately midway between the second cavity grids during the retarding phase.

Thus the kinetic energy is transferred from the electrons to the field of the second cavity. The electrons then emerge from the second cavity with reduced velocity and finally terminate at the collector.

Fig 5.3 below shows the schematic diagram of a two-cavity klystron amplifier.

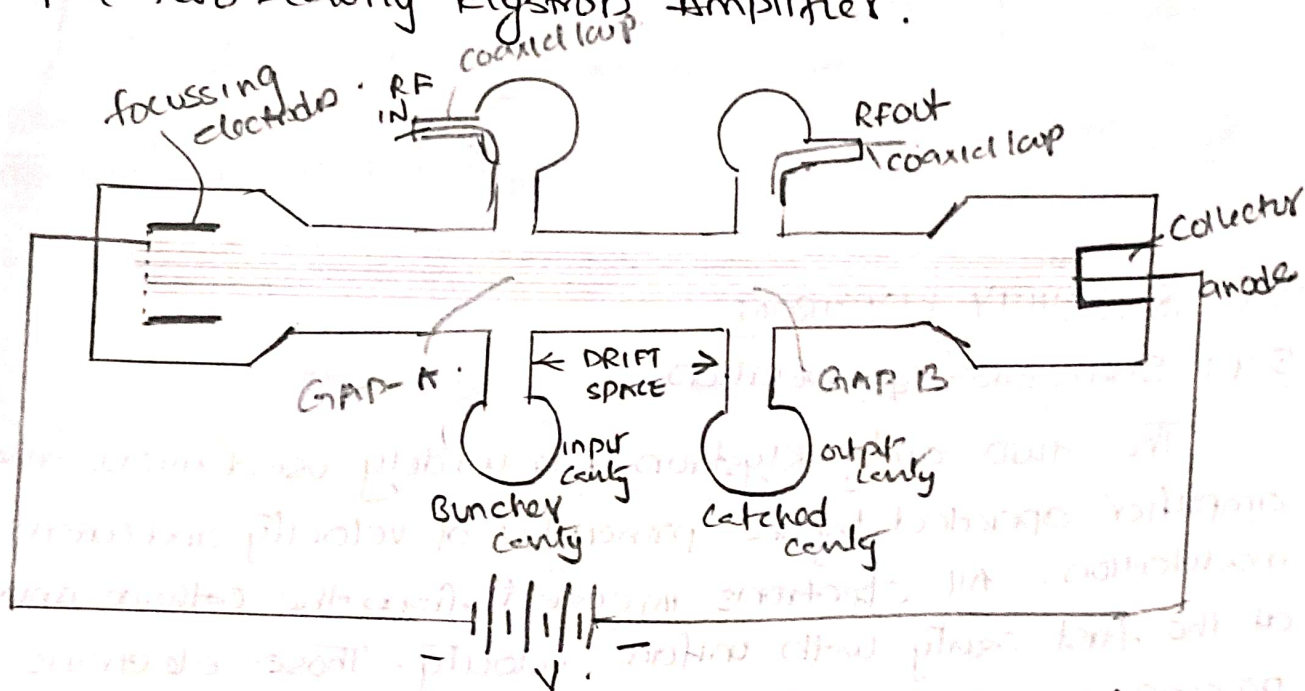


FIG 5.3. Two cavity klystron amplifier.

The cavity closer to the cathode is known as the buncher cavity or input cavity, which velocity modulates the electron beam. The other cavity is called the catcher cavity or output cavity; it catches energy from the bunched electron beam. The beam then passes through the catcher cavity and is terminated at the collector.

Performance characteristics

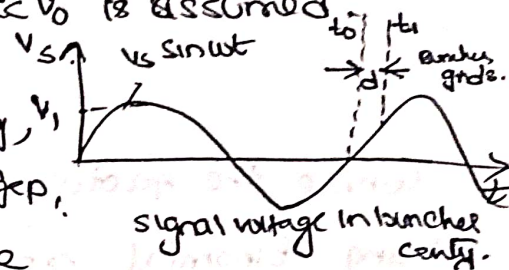
1. Frequency : 250 MHz to 100 GHz.
2. Power : 10 kW - 500 kW (CW) 30 mW (pulsed).
3. Efficiency - about 40%.
4. power gain : 15 dB - 70 dB.
5. Noise Fig : 15 - 20 dB.

Velocity modulation Process.

2- cavity klystron.

(5)

- When electrons are 1st accelerated by the high d.c voltage V_0 before entering the buncher grids, their velocity is uniform: $V_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0} \text{ m/s}$ — (1)
- When a microwave signal is applied to the input terminals, the gap voltage between the buncher grids appears as, $V_s = V_1 \sin(\omega t)$ — (2).

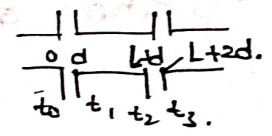


- where V_1 is the amplitude of the signal and $V_1 \ll V_0$ is assumed.
- In order to find the modulated velocity in the buncher cavity, in terms of t_0 and t_1 and gap transit angle θ_g , we need to find the avg voltage in the buncher gap.

- Since $V_1 \ll V_0$, avg transit time through the buncher gap distance d is, $\tau = \frac{d}{V_0} = t_1 - t_0$ — (3).
- avg gap transit angle is given as, $\theta_g = \omega \tau = \omega (t_1 - t_0) = \frac{\omega d}{V_0}$ — (4).
- The avg microwave voltage in the buncher gap can be found in the following way:-

$$V_s = \frac{1}{\tau} \int_{t_0}^{t_1} V_1 \sin(\omega t) dt$$

$$= \frac{V_1}{\omega \tau} [\cos(\omega t_0) - \cos(\omega t_1)] \quad \text{--- (5)}$$



Let,

$$\omega t_0 + \frac{\omega d}{2V_0} = \omega t_0 + \frac{\theta_g}{2} = A.$$

Let,

$$\omega t_0 + \frac{\omega d}{2V_0} = \omega t_0 + \frac{\theta_g}{2} = A.$$

and

$$\frac{\omega d}{2V_0} = \frac{\theta_g}{2} = B.$$

Then using the trigonometric identity $\cos(A-B) - \cos(A+B) = 2 \sin A \sin B$.

then eq, (5) becomes,

$$V_s = V_1 \frac{\sin[\omega d / (2V_0)]}{\omega d / (2V_0)} \sin\left(\omega t_0 + \frac{\omega d}{2V_0}\right) = V_1 \frac{\sin(\theta_g/2)}{\theta_g/2} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

(6)

It is defined as,

$$B_f = \frac{\sin[\omega d / (2V_0)]}{\omega d / (2V_0)} = \frac{\sin(\theta_g/2)}{\theta_g/2} \quad \text{--- (7)}$$

B_f is called beam coupling coefficient of the input cavity gap.

→ Increasing the gap transit angle θ_g decreases the coupling between the electron beam and the buncher cavity. That is the velocity modulation of the beam for a given microwave signal is decreased. Immediately after velocity modulation, the exit velocity from the buncher gap is given by

$$v(t_1) = \sqrt{\frac{2e}{m} \left[V_0 + B_1 V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \right]}$$

$$= \sqrt{\frac{2e}{m} V_0 \left[1 + \frac{B_1 V_1}{V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \right]} \quad \text{--- (8)}$$

where the factor $B_1 V_1 / V_0$ is called the depth of velocity modulation. Using binomial expansion under the assumption of, $B_1 V_1 \ll V_0$ --- (9)

eq (8) becomes,
$$v(t_1) = V_0 \left[1 + \frac{B_1 V_1}{2 V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \right] \quad \text{--- (10)}$$

eq (10) is the equation of velocity modulation. Alternatively the equation of velocity modulation can be given by,

$$v(t_1) = V_0 \left[1 + \frac{B_1 V_1}{2 V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \right] \quad \text{--- (11)}$$

BUNCHING PROCESS

The distance from the buncher grid to the location of dense electron bunching for the electron at t_b is

$$\Delta L = v_0 (t_d - t_b) \quad \text{--- (1)}$$

Similarly, the distances for the electrons at t_a and t_c are,

$$\Delta L = v_{\min} (t_d - t_a) = v_{\min} (t_d - t_b + \pi/2\omega) \quad \text{--- (2)}$$

$$\Delta L = v_{\max} (t_d - t_c) = v_{\max} (t_d - t_b + \pi/2\omega) \quad \text{--- (3)}$$

from eq (2) or (3), the minimum and maximum velocities are,

$$v_{\min} = v_0 \left(1 - \frac{\beta_1 v_1}{2v_0} \right) \quad \text{--- (4)}$$

$$v_{\max} = v_0 \left(1 + \frac{\beta_1 v_1}{2v_0} \right) \quad \text{--- (5)}$$

substituting eq (4) and (5) in eq 2 and respectively, we get the distance,

$$\Delta L = v_0 (t_d - t_b) + \left[v_0 \frac{\pi}{2\omega} - v_0 \frac{\beta_1 v_1}{2v_0} (t_d - t_b) - v_0 \frac{\beta_1 v_1}{2v_0} \frac{\pi}{2\omega} \right] \quad \text{--- (6)}$$

and

$$\Delta L = v_0 (t_d - t_b) + \left[-v_0 \frac{\pi}{2\omega} + v_0 \frac{\beta_1 v_1}{2v_0} (t_d - t_b) + v_0 \frac{\beta_1 v_1}{2v_0} \frac{\pi}{2\omega} \right] \quad \text{--- (7)}$$

The necessary condition for those electrons at t_a , t_b and t_c to meet at the same distance ΔL is,

$$v_0 \frac{\pi}{2\omega} - v_0 \frac{\beta_1 v_1}{2v_0} (t_d - t_b) - v_0 \frac{\beta_1 v_1}{2v_0} \frac{\pi}{2\omega} = 0 \quad \text{--- (8)}$$

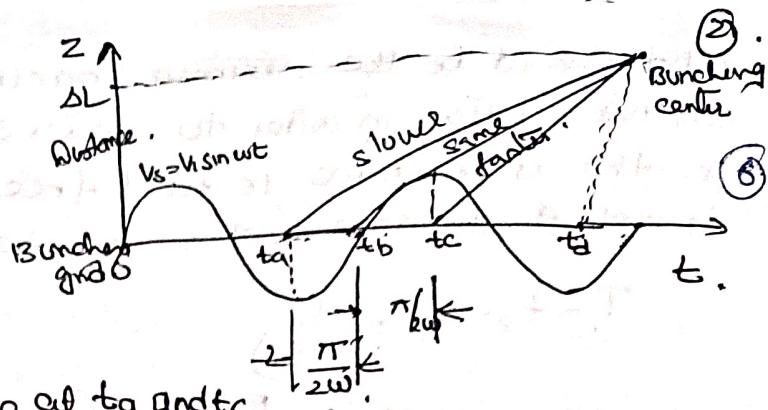
$$\text{and } -v_0 \frac{\pi}{2\omega} + v_0 \frac{\beta_1 v_1}{2v_0} (t_d - t_b) + v_0 \frac{\beta_1 v_1}{2v_0} \frac{\pi}{2\omega} = 0 \quad \text{--- (9)}$$

consequently,

$$t_d - t_b \approx \frac{\pi v_0}{\omega \beta_1 v_1} \quad \text{--- (10)}$$

$$\text{and } \Delta L = v_0 \frac{\pi v_0}{\omega \beta_1 v_1} \quad \text{--- (11)}$$

Eq (10) is not the one for a max degree of bunching. It should be noted that the mutual repulsion of the space charge is neglected.



What should be the maximum spacing between the buncher and catcher cavities in order to achieve a maximum degree of bunching. As the drift space is field-free, the transit time for an electron to travel a distance of L as shown in Fig.

$$T = t_2 - t_1 = \frac{L}{v(t_1)}$$

$$= T_0 \left[1 - \frac{\beta_1 V_1}{2V_0} \sin \left(\omega t_1 - \frac{\theta_0}{2} \right) \right] \quad (2)$$

where the binomial expansion of $(1+x)^{-1}$ for $|x| \ll 1$ has been replaced and $T_0 = L/v_0$ is the DC transit time.

In terms of radians the preceding expression can be written as

$$\omega T = \omega t_2 - \omega t_1 = \theta_0 - X \sin \left(\omega t_1 - \frac{\theta_0}{2} \right) \quad (3)$$

where, $\theta_0 = \frac{\omega L}{v_0} = 2\pi N \quad (14)$

is the transit angle b/w cavities, N is the number of electron transit cycles in the drift space, and,

$$X = \frac{\beta_1 V_1}{2V_0} \theta_0 \quad (15)$$

is defined as the bunching parameter of a klystron.

⇒ o/p power & η - The current induced by the e beam in the walls of the catcher cavity is directly proportional to the amplitude of the microwave input voltage V_1 , the fundamental component of the induced microwave current in the catcher is given by, $i_{2nd} = \beta_0 i_2 = \beta_0 2I_0 J_1(X) \cos[\omega(t_2 - \tau - T_0)] \quad (16)$

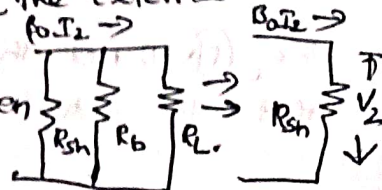
where β_0 is the coupling coefficient of the catcher gap. If the buncher and catcher cavities are identical, then $\beta_1 = \beta_0$. The fundamental component of current induced in the catcher then has a magnitude

$$i_{2nd} = \beta_0 I_2 = \beta_0 2I_0 J_1(X) \quad (17)$$

Fig shows an o/p ext circuit in which R_{sh} represents the wall resistance of catcher cavity, R_b the beam loading resistance, R_L the external load resistance, and R_{sh} the effective shunt resistance.

The o/p power delivered to the catcher cavity and load is given

$$\text{by } P_{out} = \frac{(\beta_0 I_2)^2}{2} R_{sh} = \frac{\beta_0 I_2 V_2}{2} \quad (18)$$



5.4.4. REENTRANT CAVITIES

At a frequency well below the microwave range the cavity resonator can be represented by a lumped constant resonant circuit, when the operating frequency is increased to several tens of megahertz, both the inductance and the capacitance must be reduced to a minimum in order to maintain resonance at the operating frequency, ultimately the inductance is reduced to a minimum by sheet wire. Therefore the reentrant cavities are designed for use in klystrons and microwave triodes. A reentrant cavity is one in which the metallic boundaries extend into the interior of the cavity. Several types of reentrant cavities are shown in Fig 5.6 below.

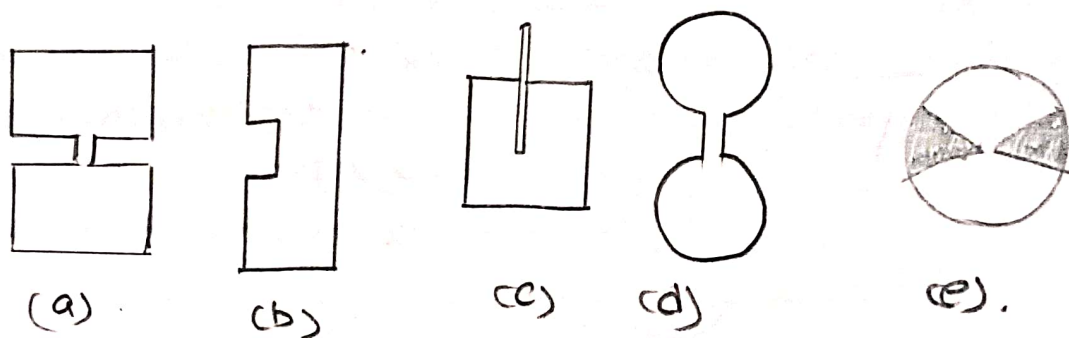


Fig 5.6 Reentrant cavities (a) Coaxial cavity (b) Radial cavity (c) Tunable cavity (d) Toroidal cavity (e) Butterfly cavity.

5.5. REFLEX KLYSTRON - STRUCTURE

The Reflex klystron is a single cavity klystron that overcomes the disadvantage of the two cavity klystron oscillator. It is a low power generator of 10 to 500 mW output at a frequency range of 1 to 25 GHz. The efficiency is about 20 to 30%. This type is widely used in the laboratory for microwave measurements and in microwave receivers as local oscillators in commercial military and airborne Doppler radars as well as missiles. The theory of the two cavity klystron can be applied to the analysis of the reflex klystron with slight modification. A schematic diagram of the reflex klystron is shown in Fig 5.7.

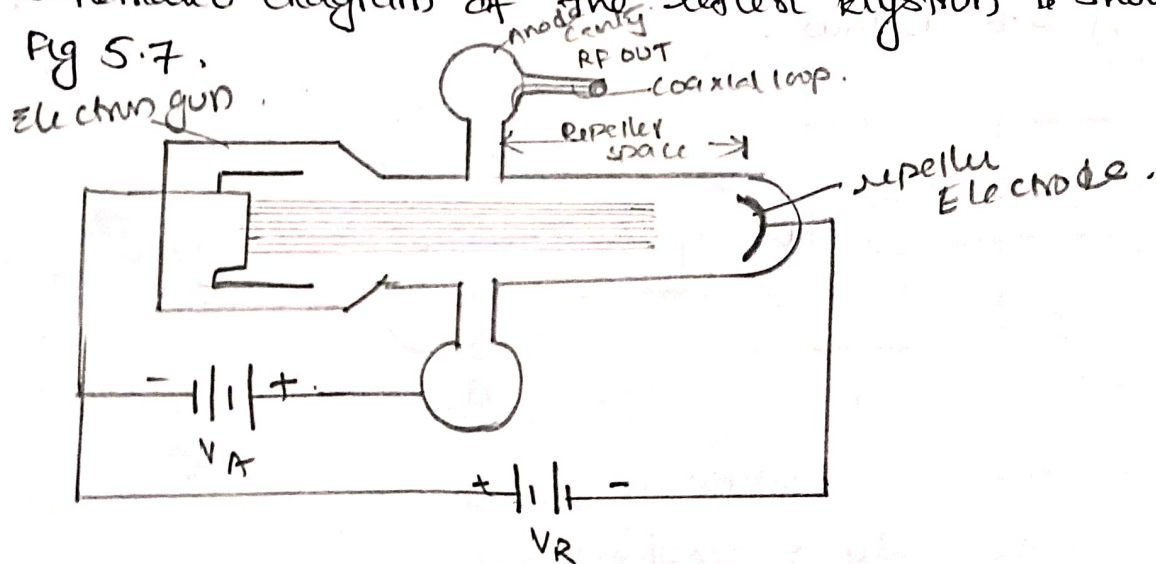


Fig 5.7 Schematic diagram of Reflex klystron.

The electron beam injected from the cathode is first velocity modulated by the cavity gap voltage. Some electrons accelerated by the accelerating field ~~and~~ enter the repeller space with greater velocity than those with unchanged velocity. Some electrons decelerated by the retarding field enter the repeller region with less velocity. All electrons turned around by the repeller voltage then pass through the cavity gap in bunches that occur once per cycle. On their return journey the bunched electrons pass through the gap during the retarding phase of the alternating

field and give up their kinetic energy to the electromagnetic energy of the field in the cavity. oscillator output energy is then taken from the cavity. The electrons are finally collected by the walls of the cavity or other grounded metal parts of the tube.

5.5.1 OSCILLATING MODES AND OUTPUT CHARACTERISTICS.

1. VOLTAGE CHARACTERISTICS.

Oscillations are obtained only for specific combinations of anode and repeller voltages that give a favorable transit time ($T = n + 3/4$). The shaded areas show possible oscillation combinations and heavy lines show optimum combinations. Each value of $n = 1, 2, 3, \dots$ is said to correspond to a different mode for the reflex klystron oscillator. The earlier the mode the larger the output power, but the voltages required are also higher as shown in Fig 5.8(a) leading to insulation problems and the possibility of lower efficiencies. As a result the modes corresponding to $n=2$ or $n=3$ are most widely used.

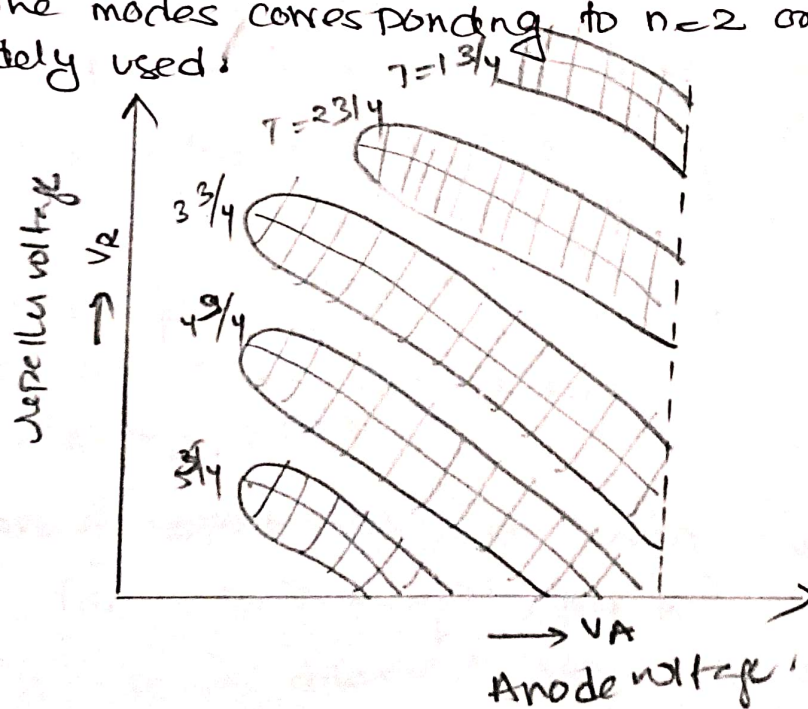


Fig 5.8(a) Voltage characteristics of Reflex klystron

2. Power output and Frequency characteristics.

The mode curves and frequency characteristics are shown in Fig 5.8(b). The Frequency of resonance of the cavity decides the frequency of oscillation. Variations of repeller voltage slightly changes the frequency. This amounts to electronic tuning of reflex klystron and thus it is also shown in Fig 5.8(b). This makes it possible to use reflex klystron as a voltage tuned oscillator or frequency modulated oscillator.

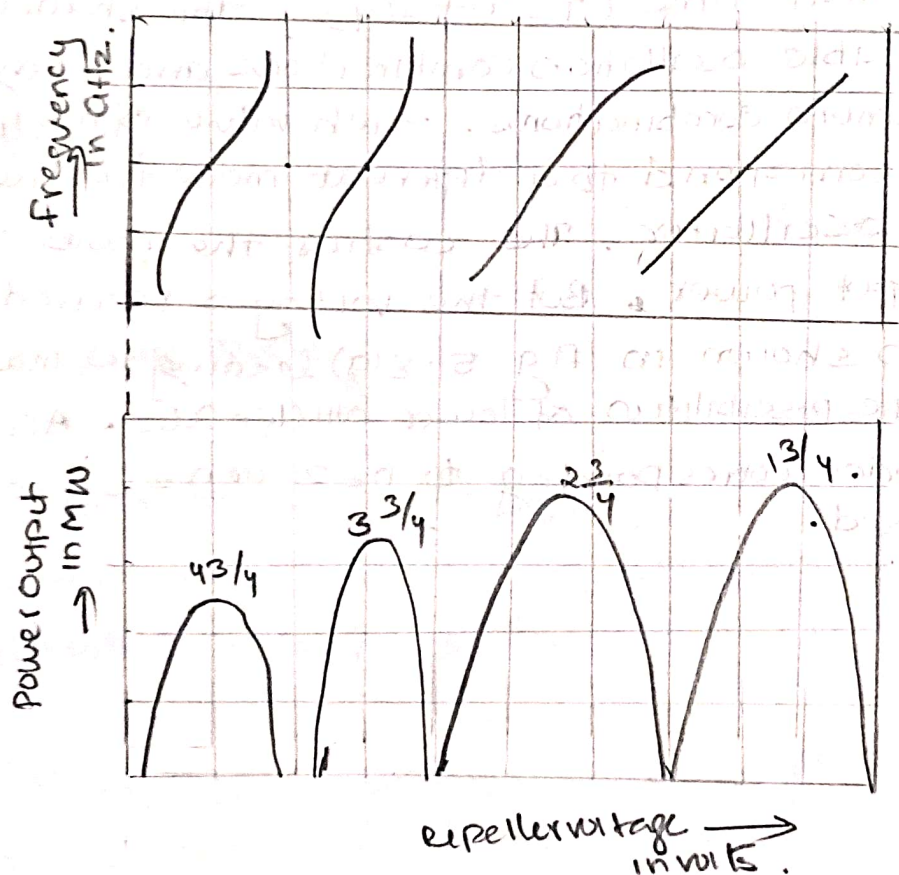


Fig 5.8(b) Power output and frequency characteristics of reflex klystron.

REFLEX KLYSTRON.

Power o/p & efficiency.

(1)

In order for a beam to generate a maximum amount of energy to the oscillation, the returning electron beam must cross the cavity gap when the gap field is maximum retarding. In this way, a maximum amount of kinetic energy can be transferred from the returning electron to the cavity walls. If ~~beam~~ for a maximum energy transfer, the round trip transit angle, referring to the center of the bunch, must be given by,

$$\omega(t_2 - t_1) = \omega T_0' = \left(n - \frac{1}{4}\right) 2\pi = N 2\pi = 2\pi n - \frac{\pi}{2} \quad (1)$$

where $V_K < V_0$ is assumed, $n =$ any +ve integer for cycle numbers, $N = n - 1/4$ is the no of modes. The bunching parameter x is same as 2-cavity klystron.

→ The beam current injected into the cavity gap from the repeller region flows in the $-ve z$ direction. So, the beam current of a reflex klystron oscillator can be written as,

$$i_2 = -I_0 - \sum_{n=1}^{\infty} 2 I_0 J_n(x') \cos[n(\omega t_2 - \phi_0' - \phi_g)] \quad (2)$$

$\phi_0' =$ round trip detuning angle.

→ The fundamental component of the current induced in the cavity by the modulated e beam is given by,

$$i_2 = -B_1 I_2 = 2 I_0 B_1 J_1(x') \cos(\omega t_2 - \phi_0') \quad (3)$$

In which ϕ_g has been neglected as a small quantity compared with ϕ_0' . The magnitude of the fundamental component is,

$$I_2 = 2 I_0 B_1 J_1(x') \quad (4)$$

The dc power supplied by the beam voltage V_0 is

$$P_{dc} = V_0 I_0 \quad (5)$$

and the ac power delivered to the load is given by,

$$P_{ac} = \frac{V_1 I_2}{2} = V_1 I_0 B_1 J_1(x') \quad (6)$$

$$\phi_0' = \omega T_0' \quad (i)$$

$$x' = \frac{B V_1}{2 V_0} \phi_0' \quad (ii)$$

bunching parameter

from eq (i) and (ii) and ①, the ratio of v_1 over v_0 is expressed by, $\frac{v_1}{v_0} = \frac{2x'}{\beta_1(2\pi n - \pi/2)}$ — (7)

substituting eq (7) in (6) yields the power o/p as,

$$P_{ac} = \frac{2V_0 I_0 x' J_1(x')}{2\pi n - \pi/2}$$

Therefore the electron efficiency of a RK oscillator is defined as, $\eta = \frac{P_{ac}}{P_{dc}} = \frac{2x' J_1(x')}{2\pi n - \pi/2}$

The factor $x' J_1(x')$ reaches a max value of 1.25 at $x' = 2.408$ and $J_1(x') = 0.52$. In practice the mode of $n=2$ has the most power output. If $n=2$ or $3/4$ mode, the maximum efficiency becomes,

$$\text{efficiency} = \frac{2(2.408) J_1(2.408)}{2\pi(2) - \pi/2} = 22.7\%$$

The maximum theoretical efficiency of a reflex klystron oscillator ranges from 20 to 30%.

REFLEX KLYSTRON

The analysis of a reflex klystron is similar to that of a 2-cavity klystron. For simplicity, the effect of space charge forces on the electron motion will again be neglected. The electron entering the cavity gap from the cathode at $z=0$ and time t_0 is assumed to have uniform velocity.

$$v_0 = 0.593 \times 10^6 \sqrt{V_0} \quad \text{--- (1)}$$

d = distance of gap
 L = distance to the gap from the cathode

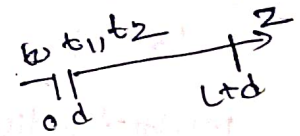
The same electron leaves the cavity gap at $z=d$ at time t_1 with velocity,

$$v(t_1) = v_0 \left[1 + \frac{\beta_1 V_1}{2V_0} \sin(\omega t_1 - \frac{\theta_0}{2}) \right] \quad \text{--- (2)}$$

This expression is same as eq (7) of 2 cavity klystron. The same electron is forced back to the cavity $z=d$ and time t_2 by the retarding electric field E , which is given by,

$$E = \frac{V_r + V_0 + V_1 \sin(\omega t)}{L} \quad \text{--- (3)}$$

This retarding field E is assumed to be constant in the z direction. The force equation for one electron in the repeller region is,

$$m \frac{d^2 z}{dt^2} = -eE = -e \frac{V_r + V_0}{L} \quad \text{--- (4)}$$


where $E = -\nabla V$ is used in the z direction only, V_r is the magnitude of the repeller voltage, and $|V_1 \sin \omega t| \ll (V_r + V_0)$ is assumed.

Integration of eq (4) twice yields

$$\frac{dz}{dt} = \frac{-e(V_r + V_0)}{mL} \int_{t_1}^t dt = \frac{-e(V_r + V_0)}{mL} (t - t_1) + K_1 \quad \text{--- (5)}$$

at $t = t_1$, $\frac{dz}{dt} = v(t_1) = K_1$; then,

$$z = \frac{-e(V_r + V_0)}{mL} \int_{t_1}^t (t - t_1) dt + v(t_1) \int_{t_1}^t dt$$

$$z = \frac{-e(V_r + V_0)}{2mL} (t - t_1)^2 + v(t_1)(t - t_1) + K_2$$

t_0 = time for electron entering gap at $z=0$
 t_1 = " " " " beam gap at $z=d$
 t_2 = " " " " returned by retarding field $z=d$ and collected on wall of center

at $t = t_1$, $z = d = k_2$; then,

$$z = \frac{-e(V_r + V_0)}{2mL} (t - t_1)^2 + v(t_1)(t - t_1) + d \quad \text{--- (6)}$$

on the assumption that the electron leaves the cavity gap at $z = d$ and time t_1 with a velocity of $v(t_1)$ and returns to the gap at $z = d$ and time t_2 , then at $t = t_2$, $z = d$,

$$0 = \frac{-e(V_r + V_0)}{2mL} (t_2 - t_1)^2 + v(t_1)(t_2 - t_1)$$

The round trip transit time in the repeller region is given by,

$$T' = t_2 - t_1 = \frac{2mL}{e(V_r + V_0)} v(t_1) = T_0' \left[1 + \frac{\beta_0 V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_0}{2}\right) \right] \quad \text{--- (7)}$$

where,

$$T_0' = \frac{2mL V_0}{e(V_r + V_0)} \quad \text{--- (8)}$$

is the round trip dc transit time of the centre of the bunch electron.

multiplication of eq (7) through by a radian frequency results in,

$$\omega(t_2 - t_1) = \theta_0' + x' \sin\left(\omega t_1 - \frac{\theta_0}{2}\right) \quad \text{--- (9)}$$

where, $\theta_0' = \omega T_0' \quad \text{--- (10)}$

is the round trip dc transit angle of the centre of the bunch electron and,

$$x' = \frac{\beta_0 V_1}{2V_0} \theta_0' \quad \text{--- (11)}$$

is the bunching parameter of the reflex klystron oscillator.

MATHEMATICAL ANALYSIS OF 2-cavity klystron. ⑤

5.4.2 → VELOCITY MODULATION PROCESS.

When the electrons are first accelerated by the high DC voltage V_0 before entering the buncher grids, their velocity is uniform:

$$V_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^8 \sqrt{V_0} \text{ m/s} \quad \text{--- (1)}$$

In eq (1), it is assumed that electrons leave the cathode with zero velocity. When a microwave signal is applied to the input terminal, the gap voltage between the buncher grids appears as,

$$V_s = V_1 \sin(\omega t) \quad \text{--- (2)}$$

where V_1 is the amplitude of the signal and $V_1 \ll V_0$ is assumed.

In order to find the modulated velocity in the buncher cavity in terms of either the entering time t_0 or the exiting time t_1 and the gap transit angle θ_g as shown fig 5.4

It is necessary to determine the average microwave voltage in the buncher gap as shown in the

Fig 5.4

Since $V_1 \ll V_0$, the average transit time through the buncher gap

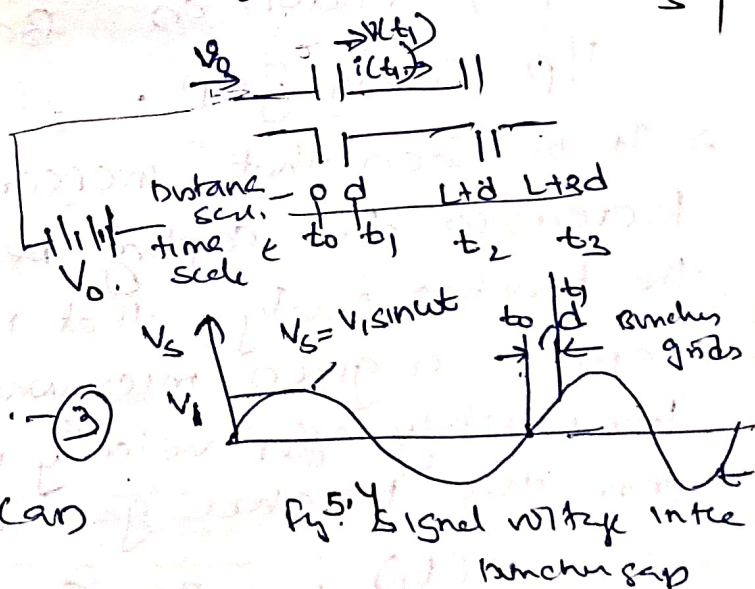
distance d is $\tau \approx \frac{d}{V_0} = t_1 - t_0$ --- (3)

The average gap transit angle can be expressed as,

$$\theta_g = \omega \tau = \omega(t_1 - t_0) = \frac{\omega d}{V_0} \quad \text{--- (4)}$$

The average microwave signal voltage in the buncher gap can be found in the following way:-

$$\langle V_s \rangle = \frac{1}{\tau} \int_{t_0}^{t_1} V_1 \sin(\omega t) dt = -\frac{V_1}{\omega \tau} [\cos(\omega t_1) - \cos(\omega t_0)]$$



$$= \frac{V_1}{\omega \epsilon} \left[\cos(\omega t_0) - \cos\left(\omega_0 + \frac{\omega d}{v_0}\right) \right] \quad \text{--- (5)}$$

Let, $\omega t_0 + \frac{\omega d}{2v_0} = \omega t_0 + \frac{\theta_g}{2} = A$.

and $\frac{\omega d}{2v_0} = \frac{\theta_g}{2} = B$.

Then using the trigonometry identity that $\cos(A-B) - \cos(A+B) = 2 \sin A \sin B$, then eq (5) becomes,

$$\begin{aligned} V_s &= V_1 \frac{\sin(\omega d / (2v_0))}{\omega d / (2v_0)} \sin\left(\omega t_0 + \frac{\omega d}{2v_0}\right) \\ &= V_1 \frac{\sin(\theta_g / 2)}{\theta_g / 2} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \quad \text{--- (6)} \end{aligned}$$

It is defined as,

$$B_i = \frac{\sin[\omega d / (2v_0)]}{\omega d / (2v_0)} = \frac{\sin \theta_g / 2}{\theta_g / 2} \quad \text{--- (7)}$$

B_i is known as Beam Coupling Coefficient of the π cavity gap.

→ It is seen that, increasing the gap transit angle θ_g , decreases the coupling between the electron beam and the buncher cavity, that is, the velocity modulation of the beam for a given microwave signal is decreased.

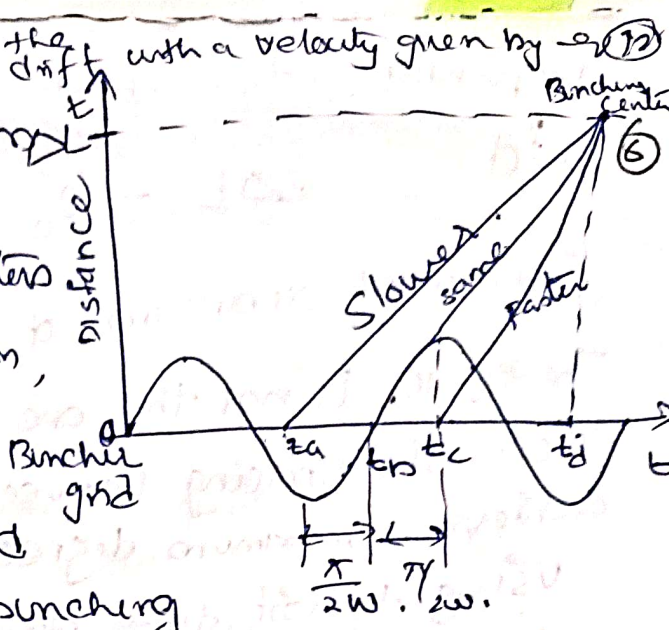
Immediately after velocity modulation, the exit velocity from the buncher gap is given by,

$$\begin{aligned} v(t_1) &= \sqrt{\frac{2e}{m} \left[V_0 + B_i V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \right]} \\ &= \sqrt{\frac{2e}{m} V_0 \left[1 + \frac{B_i V_1}{V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \right]} \quad \text{--- (8)} \end{aligned}$$

where the factor $B_i V_1 / V_0$ is called the depth of velocity modulation.

5.4.3 **APPROPRIATE ASSUMPTIONS** Since the E_s leave the buncher cavity, they drift with a velocity given by v_0 .

Bunching process: At a distance of ΔL along the beam from the buncher cavity, the beam electrons have drifted into dense clusters. Fig shows, the trajectories of minimum, zero, and maximum electron acceleration.



→ The distance from the buncher grid to the location of dense electron bunching for the electron at t_b is

Fig. 5.5

$$\Delta L = v_0(t_d - t_b) \quad \text{--- (1)}$$

Similarly distance for t_a and t_c are

$$\Delta L = v_{min}(t_d - t_a) = v_{min}(t_d - t_b + \frac{\pi}{2w}) \quad \text{--- (2)}$$

$$\Delta L = v_{min}(t_d - t_c) = v_{min}(t_d - t_b + \frac{\pi}{w}) \quad \text{--- (3)}$$

From eq. 10 & 11, the minimum and maximum velocities are,

$$v_{min} = v_0 \left(1 - \frac{\beta_0 V_1}{2V_0} \right) \quad \text{--- (4)}$$

$$v_{max} = v_0 \left(1 + \frac{\beta_0 V_1}{2V_0} \right) \quad \text{--- (5)}$$

Substitute eq 4 and 5 in eq 2 and 3, respectively,

~~$$\Delta L = v_{min}(t_d - t_a) = v_0(t_d - t_b + \frac{\pi}{2w}) + \left[v_0 \frac{\pi}{2w} - v_0 \frac{\beta_0 V_1}{2V_0} (t_d - t_b) - v_0 \frac{\beta_0 V_1}{2V_0} \cdot \frac{\pi}{2w} \right]$$~~

$$\Delta L = v_0(t_d - t_b) + \left[v_0 \frac{\pi}{2w} - v_0 \frac{\beta_0 V_1}{2V_0} (t_d - t_b) - v_0 \frac{\beta_0 V_1}{2V_0} \cdot \frac{\pi}{2w} \right]$$

and

$$\Delta L = v_0(t_d - t_b) + \left[-v_0 \frac{\pi}{2w} + v_0 \frac{\beta_0 V_1}{2V_0} (t_d - t_b) + v_0 \frac{\beta_0 V_1}{2V_0} \cdot \frac{\pi}{2w} \right]$$

The necessary condition for those electrons at t_a , t_b and t_c to meet at the same distance ΔL is,

$$v_0 \frac{\pi}{2w} - \frac{v_0 \beta_0 V_1}{2V_0} (t_d - t_b) - v_0 \frac{\beta_0 V_1}{2V_0} \cdot \frac{\pi}{2w} = 0 \quad \text{--- (6)}$$

consequently, $t_d - t_b \approx \frac{\pi V_0}{\omega B_i V_i}$ — (9)

and $\Delta L = V_0 \cdot \frac{\pi V_0}{\omega B_i V_i}$ — (10)

The mutual repulsion of the space charge is neglected. The eq 10 is not the one for a maximum degree of bunching. Then the spacing between the buncher and catcher cavity to achieve a maximum degree of bunching is obtained by using transit time for an electron to travel a distance of L as shown in Fig, is given by,

$T = t_2 - t_1 = \frac{L}{V(t_1)} = T_0 \left[1 - \frac{B_i V_i}{2V_0} \sin\left(\omega t_1 - \frac{\theta_0}{2}\right) \right]$ — (11)

where the binomial expansion of $(1+x)^{-1}$ for $|x| \ll 1$ has been replaced and $T_0 = L/V_0$ is the dc transit time.

In terms of radians the preceding expression can be written as,

$\omega T = \omega t_2 - \omega t_1 = \theta_0 - X \sin\left(\omega t_1 - \frac{\theta_0}{2}\right)$ — (12)

where, $\theta_0 = \frac{\omega L}{V_0} = 2\pi N$, is the dc transit angle between cavities, N is the number of electron transit cycles in the drift space, and $X = \frac{B_i V_i}{2V_0} \theta_0$,

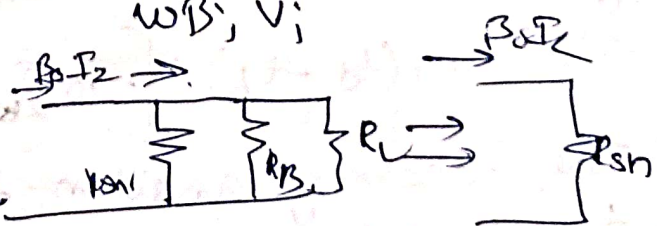
is the bunching parameter of a klystron.

The fundamental component I_f has its maximum amplitude at $X = 1.841$

The optimum length $L_{\text{optimum}} = \frac{3.682 V_0 V_0}{\omega B_i V_i}$

o/p power and efficiency

$P_{\text{out}} = \left(\frac{B_0 I_2}{2} \right)^2 R_{\text{sh}} = \frac{B_0 I_2 V_2}{2}$



$\frac{P_{\text{out}}}{P_{\text{in}}} = \frac{B_0 I_2 V_2}{2 I_0 V_0}$

6.1 HELIX TWT'S.

INTRODUCTION

The travelling wave tube is a new type of tube which has displayed considerable promise as a broad band amplifier. TWT's are broad band devices in which there are no cavity resonators. The interaction space in a TWT is extended and the electron beam exchanges energy ~~there~~ with the RF wave over the full length of the tube. The TWT makes use of distributed interaction between an electron beam and a travelling wave. To prolong the interaction between an electron beam and the RF field it is necessary to ensure that they are both travelling in the same direction with nearly the same velocity. Thus it differs from the klystron in which the electron beam travels and the RF field remains stationary.

The electron beam travels with a velocity governed by the anode voltage. The RF field propagates with a velocity equal to velocity of light V_0 . The interaction between the RF field and the moving electron beam will take place only when the RF field is retarded by some means. Normally the slow wave structures are utilised to retard the RF field, like helix-~~o~~.

6.2 TYPES and characteristics of slow wave structures.

The slow wave structures are special circuits that are used in microwave tubes to reduce the wave velocity in a certain direction so that the electron beam and the signal wave can interact. The

The various types of slow wave structures are:-(fig 6.1)

1. Helical line
2. Folded back line
3. Zigzag line
4. Interdigital line
5. Combed waveguide.

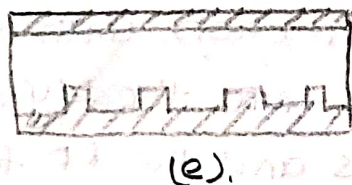
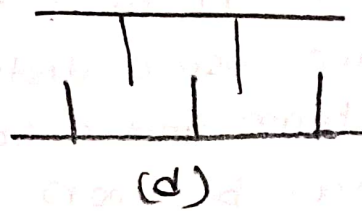
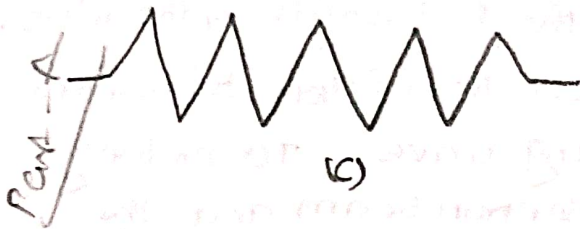
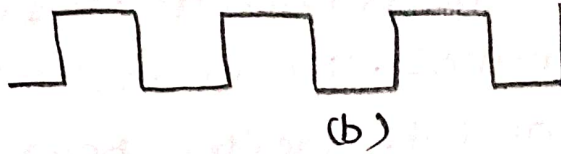
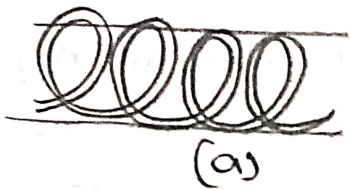


Fig 6.1 slow wave structures (a) Helical line (b) Folded back line (c) zigzag line (d) Interdigital line (e) Combed w.g.

The phase velocity of a wave in ordinary waveguides is greater ~~than~~ than velocity of light in a vacuum. In the operation of travelling wave and magnetron type devices, the electron beam must keep in step with the microwave signal. Since the electron beam can be accelerated only to velocities that are about a fraction of the velocity of light, a slow wave structure must be incorporated in the microwave devices so that the phase velocity of the microwave signal can keep pace with that of the electron beam for effective interaction.

The commonly used slow wave structure is a helical coil with a concentric conducting cylinder.

6.3 STRUCTURE OF TWT AND AMPLIFICATION PROCESS.

The physical construction of a typical TWT along with the schematic electrode arrangement is shown in Fig 6.2. It has an electron gun as used in klystrons, which is used to produce a narrow constant velocity electron beam. This electron beam is in turn passed through the centre of a long axial helix. A magnetic focussing field is provided to prevent the beam from spreading and to guide it through the centre of the helix. Helix is a loosely wound thin conducting helical wire, which acts as a slow wave structure.

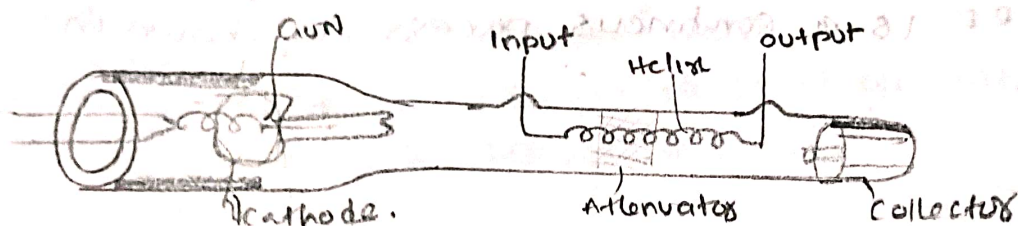


Fig 6.2. (a)

The signal to be amplified is applied to the end of the helix adjacent to the electron gun. The amplified signal appears at the output or other end of the helix under appropriate operating conditions.

Operation:- When the applied RF signal propagates around the turns of the helix, it produces an electric field at the centre of helix. The RF field propagates with the velocity of light. The axial electric field due to RF signal travels with the velocity of light multiplied by the ratio of helix pitch to the helix circumference. When the velocity of the electron beam travelling through the helix approximates the rate of advance of the axial field, then interaction takes place between them in such a way that on an

average the electron beam delivered energy to the RF wave on the helix.

Thus, the signal wave grows and amplified output is obtained at the output of the TWT. The axial phase velocity v_p is given by,

$$v_p = v_c (\text{Pitch}/2\pi r) \text{ --- (1)}$$

where r is the radius of the helix and is essentially constant over a range of frequencies and this characteristic of helix slow wave structure enables TWT to have broad band operation. Helix since provides the least change in v_p with freq, is preferred over other slow wave structures for TWT. The energy increase in the RF is a continuous process as shown in Fig 6.3.

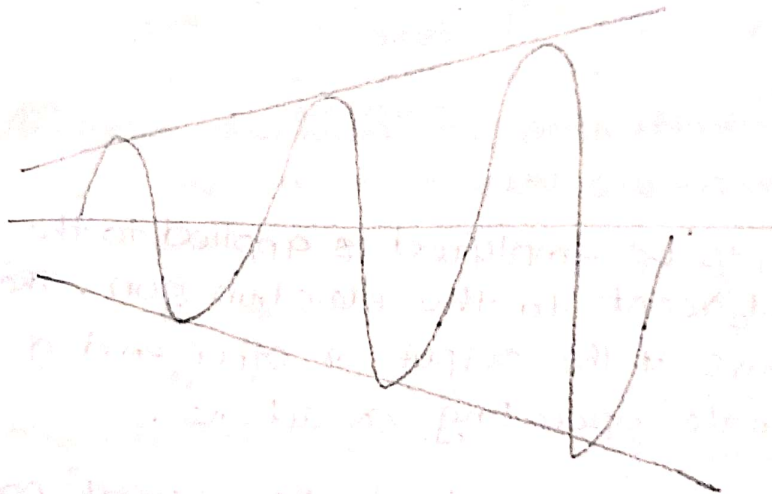


Fig 6.3. Amplification process.

6.4.

SUPPRESSION OF OSCILLATIONS

6.5 GAIN CONSIDERATIONS.

The Gain parameter of TWT is defined as,

$$G = \left(\frac{I_{dc} Z_0}{4 V_{dc}} \right)^{1/3}.$$

output power Gain, A_p is defined as,

$$A_p = 10 \log \left| \frac{V(L)}{V(0)} \right|^2$$

$$= -9.54 + 47.32 LC \text{ dB.}$$

here, LC is numerical number.

L = circuit length.

C = gain parameter,

$V(L)$ = the amplitude of o/p voltage

$V(0)$ = voltage at $z=0$

PROBLEMS.

Q. The parameters of a TWT are beam voltage, $V_{dc} = 2.5 \text{ kV}$, Beam current, $I_{dc} = 25 \text{ mA}$. The characteristic impedance of the helix $Z_0 = 10 \Omega$. It is operated at a frequency of $f = 9.5 \text{ GHz}$, circuit length = 40. Find out the gain parameter, output power gain & P_e .

Sol. The gain parameter, $G = \left(\frac{I_{dc} Z_0}{4 V_{dc}} \right)^{1/3}$.

$$= \left(\frac{25 \times 10^{-3} \times 10}{4 \times 2.5 \times 10^3} \right)^{1/3} = \left(\frac{10^{-4}}{4} \right)^{1/3}$$

$$G = 0.0292.$$

The output power gain

$$A_p = -9.54 + 4.73 LC$$

$$= -9.54 + 47.3 \times 40 \times 0.0292$$

$$A_p = 45.7 \text{ dB}$$

$$\beta_c = \frac{\omega}{V_c}$$

$$\text{Here, } \omega = 2\pi f, f = 9.5 \text{ GHz}$$

$$V_{dc} = 0.593 \times 10^6 \sqrt{V_c}$$

$$V_{dc} = 2.5 \times 10^3 \text{ V}$$

$$\text{i.e. } \beta_c = \left(\frac{2\pi \times 9.5 \times 10^9}{0.593 \times 10^6 \sqrt{2.5 \times 10^3}} \right) = 2.01 \times 10^3 \text{ rad/m}$$

Q. A helical TWT has dimensions of 2 mm with 50 turns per cm. (a) calculate axial phase velocity and the anode voltage at which the TWT can be operated for useful gain.

$$\text{Sol. } v_p = \text{velocity of light} \times \frac{\text{Pitch}}{\text{Circumference}}$$

$$\text{Pitch} = \frac{1}{50 \text{ cm}} = 0.02 \text{ cm} = 2 \times 10^{-4} \text{ m}$$

$$\text{Circumference} = \pi \times D = \pi \times 2 \times 10^{-3} \text{ m} = 6.284 \times 10^{-3} \text{ m}$$

$$v_p = 3 \times 10^8 \times 2 \times \frac{10^4}{6.284 \times 10^{-3}}$$

$$eV_0 = \frac{1}{2} m v_p^2$$

$$V_0 = \frac{1}{2} \frac{m}{e} v_p^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (0.954 \times 10^7)^2 \times \frac{1}{1.6 \times 10^{-19}}$$

$$= \underline{\underline{25.92 \text{ kV}}}$$

6.5M TYPE TUBES

6.5.1 INTRODUCTION.

In the M-TYPE TUBES, the dc magnetic field is perpendicular to the dc electric field. These are also called crossed field devices. In all crossed field devices the dc magnetic field plays a direct role in the RF interaction process. They are called M type tubes after the French term *tubes à propagation des ondes à champs magnétique* i.e tubes ~~for~~ for propagation of waves in a magnetic field. The principle tube in this M type is the MAGNETRON.

6.5.2. MAGNETRON - DIFFERENT TYPES.

There are 3 types of magnetrons :-

1. Negative Resistance type - They make use of negative resistance between two anode segments but have low efficiency and are very useful at low frequencies ($< 500 \text{ MHz}$).
2. Cyclotron frequency magnetron - depend on synchronism between an alternating component of electric and periodic oscillation of electrons in a direction parallel to this field.
3. Cavity magnetron - depend upon the interaction of electrons with a rotating electro-magnetic field of constant angular velocity.

6.5.3. CYLINDRICAL MAGNETRON.

A schematic diagram of a cylindrical magnetron oscillator is shown in Fig 6.5, this type of magnetron is also called a conventional magnetron.

In a cylindrical magnetron, several resonant cavities are connected to the gaps. The dc voltage is

is applied between the cathode and anode. The magnetic flux density B_0 is in the positive z direction. When the dc voltage and the magnetic flux density B_0 is the positive z direction. When the dc voltage and the magnetic flux are adjusted properly, the electrons will follow cycloidal paths in the cathode anode space under the combined force of both electric and magnetic fields as shown in 6.5.1.

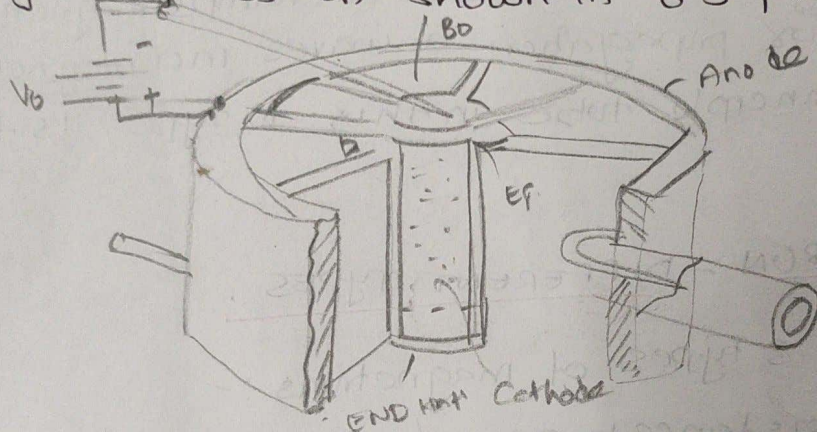


Fig 6.5 Schematic diagram of a cylindrical magnetron.

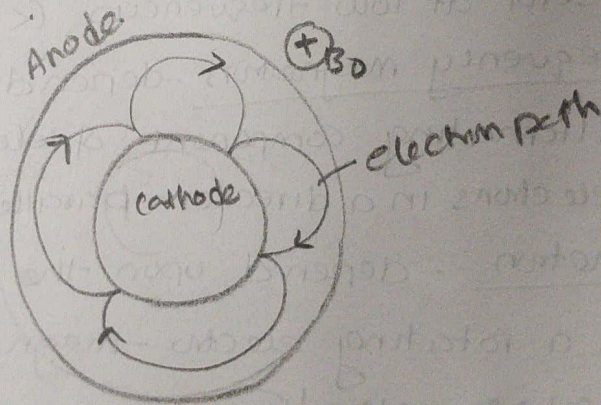


Fig 6.5.1 Electron path in a cylindrical magnetron.

Operation:- To understand the operation of the magnetron, we must first look at how the electrons behave in the presence of crossed electric and magnetic fields. In the absence of magnetic field the electron emitted from cathode moves along the radial direction as shown by 'p', in the fig 6.5.2 on

accent of the radial anode to cathode electric field.
 → When a small magnetic field B is applied at right angle to radial electric field the electron trajectory bends and follows a curved path as shown by curve q in Fig 6.5.2. The Force exerted on the electron by the magnetic field is given by $F = -e(v \times B)$. As the magnetic field is increased the magnetic force exerted on electron increases and the radius of curvature of electron path decreases.

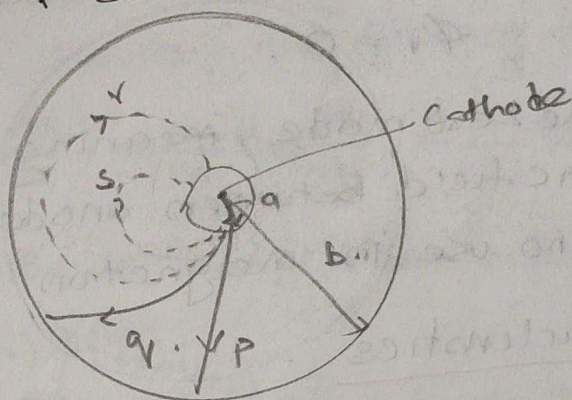


Fig. electron trajectories at various magnetic fields.

6.5.4

MODES OF RESONANCE & PI MODE OPERATIONS

In general a N -cavity tightly coupled system will have N -modes of operation each of which is uniquely characterised by a combination of frequency and phase of oscillation relative to the adjacent cavity. In addition, those modes must be self consistent so that the total phase shift around the ring of cavity resonators is $2n\pi$ where n is an integer. For example, a phase ϕ should be 40° between centres

of a 8-cavity magnetron will mean that the first cavity is out of phase with itself by 320° . The correct ~~mean~~ minimum phase shift should be 45° ($45 \times 8 = 360^\circ$)

Therefore if ϕ_0 represents the relative phase change of the ac electric field across adjacent cavities, then

$$\phi_0 = \frac{2\pi n}{N} \text{ where } n = 0, \pm 1, \pm 2, \dots, \pm(N/2 - 1), \pm \frac{N}{2}$$

i.e. $N/2$ mode of resonance can exist if N is an even number.

If $n = \frac{N}{2}$, $\phi_0 = \pi$

This mode of resonance is called π -mode.

If $n = 0$, $\phi_0 = 0$.

This is the zero mode, meaning there will be no RF electric field between anode & cathode and is of no use in magnetron operation.

6.5.3 Output characteristics

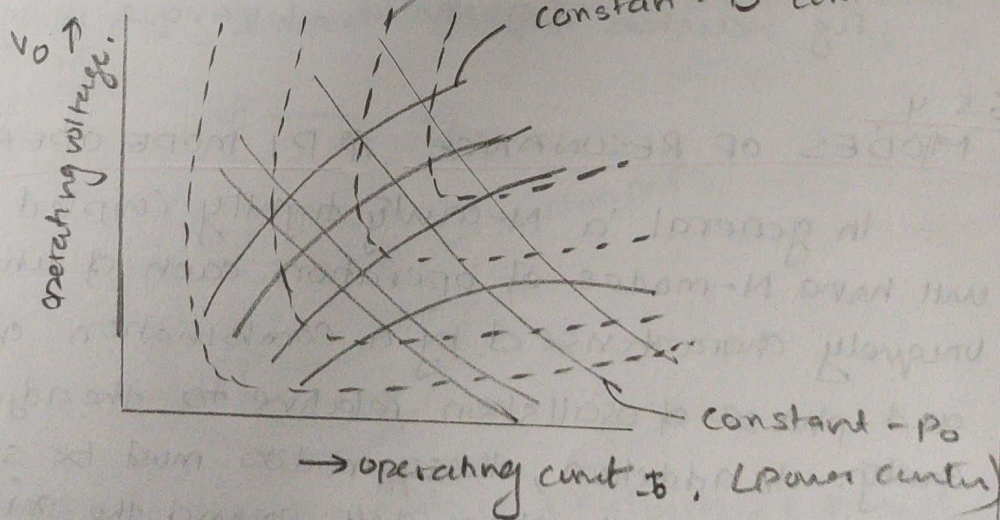


Fig: 6.5.3 - performance chart of magnetron.

The characteristics of magnetron gives contours of constant power output, constant efficiency and constant magnetic field strength as function of the dc anode voltage and current, as shown in fig 6.5.3.

HULL CUT OFF CONDITION

→ The equation of motion for electrons in a cylindrical magnetron can be written with the aid of equation of electrons in magnetic field in a cylindrical co-ordinates as:-

$$\frac{d^2 r}{dt^2} - r \left(\frac{d\phi}{dt} \right)^2 = \frac{e}{m} E_r - \frac{e}{m} r B_z \frac{d\phi}{dt} \quad \text{--- (1)}$$

$$\frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) = \frac{e}{m} B_z \frac{dr}{dt} \quad \text{--- (2)}$$

where $\frac{e}{m} = 1.759 \times 10^{11}$ C/kg is the charge to mass ratio of the electron and $B_0 = B_z$ is assumed in the positive z direction.

→ Rearranging eq 2 results in the following form.

$$\frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) = \frac{e}{m} B_z \frac{dr}{dt} = \frac{1}{2} \omega_c \frac{d}{dt} (r^2) \quad \text{--- (3)}$$

where, $\omega_c = \frac{e}{m} B_z$ is the cyclotron angular frequency. Integration of eq (3) yields,

$$r^2 \frac{d\phi}{dt} = \frac{1}{2} \omega_c r^2 + \text{Constant} \quad \text{--- (4)}$$

at $r=a$, where a is the radius of the cathode cylinder, and $\frac{d\phi}{dt} = 0$, $\text{Constant} = -\frac{1}{2} \omega_c a^2$. The angular velocity is expressed by,

$$\frac{d\phi}{dt} = \frac{1}{2} \omega_c \left(1 - \frac{a^2}{r^2} \right) \quad \text{--- (5)}$$

Since the magnetic field does no work on the electrons, the kinetic energy of the electron is given by,

$$\frac{1}{2} m v^2 = eV \quad \text{--- (6)}$$

However, the electron velocity has r and ϕ components such as,

$$v^2 = \frac{2eV}{m} = v_r^2 + v_\phi^2 = \left(\frac{dr}{dt} \right)^2 + \left(r \frac{d\phi}{dt} \right)^2 \quad \text{--- (7)}$$

at $r=b$, where b is the radius from the center of the cathode to the edge of the anode, $v = v_0$ and $dr/dt = 0$, when the electrons just graze the anode, eq (5) and eq (7) become,

$$\frac{d\phi}{dt} = \frac{1}{2} \omega_c \left(1 - \frac{a^2}{b^2} \right) \quad \text{--- (8)}$$

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$$b^2 \left(\frac{d\phi}{dt} \right)^2 = \frac{2e}{m} V_0 \quad \text{--- (9)}$$

Sub, of eq (8) into 9 results in

$$b^2 \left[\frac{1}{2} \omega_c \left(1 - \frac{a^2}{b^2} \right) \right]^2 = \frac{2e}{m} V_0 \quad \text{--- (10)}$$

The electron will acquire a tangential as well as a radial velocity. whether the electron will just graze the anode and return toward the cathode depends on the relative magnitudes of V_0 and B_0 .

The Hull cutoff magnetic equation is obtained from eq (10) as,

$$B_{0c} = \frac{\left(8 V_0 \frac{m}{e} \right)^{1/2}}{b \left(1 - \frac{a^2}{b^2} \right)} \quad \text{--- (11)}$$

This means that if $B_0 > B_{0c}$ for a given V_0 , the electron will not reach the anode. conversely, the cutoff voltage is given by,

$$V_{0c} = \frac{e}{8m} B_0^2 b^2 \left(1 - \frac{a^2}{b^2} \right)^2 \quad \text{--- (12)}$$

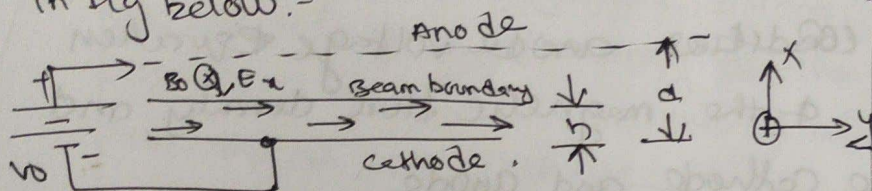
This means that if $V_0 < V_{0c}$ for a given B_0 , the electrons will not reach the anode, eq (12) is called the Hull cut off voltage equation.

HARTREE CONDITION :-

(7)

The Hull cutoff condition determines the anode voltage or magnetic field necessary to obtain non zero anode current as a function of the mag. field or anode voltage in the absence of an electromagnetic field.

The Hartree condition can be derived as follows and as shown in fig below:-



where, $B_0 = B_z$ is the magnetic flux density in the +ve z direction, V = potential.

The electron beam lies within a region extending a distance h from the cathode, where h is known as the hub thickness.

The spacing b/w the cathode and anode is d .

The electron motion is assumed to be in the positive y direction with a velocity v_y .

$$v_y = - \frac{E_x}{B_0} = \frac{1}{B_0} \frac{dV}{dx} \quad (1)$$

From the principle of energy conservation,

$$\text{we have, } \frac{1}{2} m v_y^2 = eV \quad (2)$$

Combining eq (1) and (2) yields,

$$\left(\frac{dV}{dx} \right)^2 = \frac{2eV}{m} B_0^2 \quad (3)$$

The differential equation may be rearranged as,

$$\left(\frac{m}{2eB_0} \right)^{1/2} \frac{dV}{\sqrt{V}} = dx \quad (4)$$

Integration of eq (4) yields the potential within the electron beam as,

$$V = \frac{eB_0^2}{2m} x^2 \quad (5)$$

where the constant of integration of eq (3) has been eliminated for $V=0$ at $x=0$. The potential & electric field at the hub surface are given by

$$V(h) = \frac{e}{2m} B_0^2 h^2 \quad (6)$$

and

$$E_x = - \frac{dV}{dx} = - \frac{e}{m} B_0^2 h \quad (7)$$

The potential at the anode is thus obtained from

eq (7) as

$$\begin{aligned} V_0 &= - \int_0^d E_x dx \\ &= - \int_0^h E_x dx - \int_h^d E_x dx \\ &= V(h) + \frac{e}{m} B_0^2 h(d-h) \\ &= \frac{e}{m} B_0^2 h(d-h/2) \quad (8) \end{aligned}$$

The electron velocity at the hub surface is obtained from

eq (1) and (7) as

$$v_y(h) = \frac{e}{m} B_0 h \quad (9)$$

for synchronism, this electron velocity is equal to the phase velocity of the slow wave

that is,

$$\frac{\omega}{\beta} = \frac{e}{m} B_0 h \quad \text{--- (10)}$$

For the π mode operation, the anode potential is finally given by,

$$V_{oh} = \frac{\omega B_0 d}{\beta} - \frac{m}{2e} \frac{\omega^2}{\beta^2}$$

This is the Helix ~~condition~~ anode voltage equation that is a function of the magnetic flux density and the spacing b/w the cathode and anode.

Gain considerations

The o/p power gain in decibels $A_p = 10 \log \left| \frac{V(l)}{V(0)} \right|^2$ is defined as,

$$= -9.54 + 47.3 \text{ NC dB}$$

N - length ..

C = gain parameter of the circuit

6-5-6.

PROBLEMS.

Q. A 250 kW pulsed cylindrical magnetron is operated with the following parameters; Anode voltage = 25 kV, Peak anode current = 25 A, magnetic field $B = 0.35 \text{ Wb/m}^2$, radius of cathode cylinder = 4 cm, Radius of cylinder = 8 cm.

Calculate (i) the efficiency of the magnetron (ii) the cyclotron freq ω_c . (iii) the cut off magnetic field.

iv. The cut off voltage.

Sol . efficiency $\eta = \frac{P_{out}}{P_{in}} \times 100$

$$= \frac{250 \times 10^3 \times 10^2}{25 \times 10^3 \times 25} = 40\%.$$

(ii) the cyclotron frequency $\omega_c = \frac{e}{m} B$.

$$= 1.76 \times 10^{11} \times 0.35$$

$$= 0.616 \times 10^{11} \text{ rad/s}$$

(iii) The cut off magnetic field

$$B_c = \frac{6.75 \times 10^6 \times 8 \times 10^{-2}}{(64-16) \times 10^{-4}} \sqrt{25 \times 10^5}$$

$$= 0.018 \text{ Wb/m}^2 = 18 \text{ mWb/m}^2$$

(iv) The cut off voltage = $\frac{1}{8} \eta B^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2$

$$= \frac{1}{8} \times 1.76 \times 10^{11} (0.35)^2 \times (8 \times 10^{-2})^2 \left(1 - \frac{4^2}{8^2}\right)^2$$

$$= 151.58 \text{ kV}$$

Ans

2Q. A linear magnetron has the following operation parameters:- Anode voltage: $V_0 = 15 \text{ kV}$, cathode current: $I_0 = 1.2 \text{ A}$, operating frequency: $f = 8 \text{ GHz}$, magnetic flux density: $B_0 = 0.015 \text{ Wb/m}^2$, Hub thickness: $h = 2.77 \text{ cm}$, Distance b/w anode and cathode: $d = 5 \text{ cm}$.

Calculate a) the electron velocity at the hub surface.
b) the phase velocity for synchronism c) the Hartree anode voltage

Solution:- a) The electron velocity is,

$$V = 1.759 \times 10^{11} \times 0.015 \times 2.77 \times 10^{-2} \\ = 0.73 \times 10^8 \text{ m/s},$$

b. The phase velocity is

$$V_{ph} = \frac{\omega}{\beta} = 0.73 \times 10^8 \text{ m/s}.$$

c. The Hartree anode voltage is,

$$V_{ah} = 0.73 \times 10^8 \times 0.015 \times 5 \times 10^{-2} - \frac{1}{2 \times 1.759 \times 10^{11}} \times (0.73 \times 10^8)^2 \\ = 5.475 \times 10^4 - 1.515 \times 10^4 \\ = \underline{\underline{39.60 \text{ kV}}}.$$

PROBLEMS (MAGNETRON)

- ①. An X band pulse cylindrical magnetron has the following operating parameters:-

Anode voltage $V_0 = 26 \text{ kV}$

Beam current $I_0 = 27 \text{ A}$

magnetic flux density $B_0 = 0.336 \text{ Wb/m}^2$

Radius of cathode cylinder $a = 5 \text{ cm}$

Radius of vane edge $b = 10 \text{ cm}$ to center

Calculate a) the cyclotron angular freq, (b) cut off voltage for

(c) The cut off mag flux density for fixed V_0 , fixed B_0 .

Sol: (a) The cyclotron angular frequency ω_c ,

$$\omega_c = \frac{e}{m} B_0 = 1.759 \times 10^{11} \times 0.336 = 5.91 \times 10^{10} \text{ rad/s}$$

(b) cut off voltage for a fixed B_0 is

$$V_{oc} = \frac{e}{8m} B_0^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2$$

$$= \frac{1}{8} \times 1.759 \times 10^{11} (0.336)^2 (10 \times 10^{-2})^2 \left(1 - \frac{5^2}{10^2}\right)^2$$

$$= \underline{\underline{139.50 \text{ kV}}}$$

(c) The cut off magnetic flux density for a fixed V_0 is

$$B_{oc} = \left(8 \times 26 \times 10^3 \times \frac{1}{1.759 \times 10^{11}}\right)^{1/2} \times \left[10 \times 10^{-2} \left(1 - \frac{5^2}{10^2}\right)\right]^{-1}$$

$$= \underline{\underline{14.495 \text{ mWb/m}^2}}$$

$$B_{oc} = \frac{\left(8 V_0 \frac{m}{e}\right)^{1/2}}{b \left(1 - \frac{a^2}{b^2}\right)}$$

PROBLEMS (TWT)

Q. A normal circular magnetron has the following parameters:-

Inner Radius $R_i = 0.15 \text{ m}$

outer Radius $R_o = 0.45 \text{ m}$.

mag flux density $B_o = 1.2 \text{ mwb/m}^2$

a) Determine the full cutoff voltage

b) Determine the cut off mag flux density if the beam voltage

$V_o = 6000 \text{ V}$.

c) Determine cyclotron angular freq. in GHz

Sol (a) cut off voltage $V_o = \frac{e}{8m} B_o^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2$

$$= \frac{1.759 \times 10^{11}}{8} \times (1.2 \times 10^{-3})^2 (0.45)^2 \left(1 - \left(\frac{0.15}{0.45}\right)^2\right)^2$$

$$= 50.666 \text{ kW}$$

(b) $B_{oc} = \frac{(20 \text{ me})^{1/2}}{b \left(1 - \frac{a^2}{b^2}\right)} = \sqrt{\frac{8 \times 6000}{1.759 \times 10^{11}}} \times \frac{1}{0.45 \left(1 - \frac{0.15^2}{0.45^2}\right)}$

$$= 130.595 \text{ mwb/m}^2 \quad \underline{\underline{1.3 \text{ mwb}^2/\text{m}^2 \text{ line}}}$$

(c) $\omega_c = \frac{e B_o}{m} \times \frac{1}{2\pi} = \frac{1.759 \times 10^{11} \times 1.2 \times 10^{-3}}{2\pi}$

$$= \underline{\underline{0.336 \text{ GHz}}}$$

PROBLEMS. (TWT)

- ①. A helical TWT has diameter of 2mm with 50 turns per cm.
 (a) calculate axial phase velocity and the anode voltage at which the TWT can be operated for useful gain.

Sol a) $V_p = \text{velocity of light} \times \frac{\text{Pitch}}{\text{circumference}}$

$$\text{Pitch} = \frac{1}{50 \text{ cm}} = 0.02 \text{ cm} = 2 \times 10^{-4} \text{ m.}$$

$$\text{circumference} = \pi \times D = \pi \times 2 \times 10^{-3} \text{ m} = 6.284 \times 10^{-3} \text{ m.}$$

$$V_p = 3 \times 10^8 \times 2 \times \frac{10^{-4}}{6.284 \times 10^{-3}}$$

Ans (b) $eV_0 = \frac{1}{2} m V_p^2$

$$V_0 = \frac{1}{2} \frac{m}{e} V_p^2 = \frac{1}{2} \times 9.1 \times \frac{10^{-31}}{1.6 \times 10^{-19}} \times (0.9548 \times 10^7)^2$$

$$= \underline{\underline{25.92 \text{ kV}}}$$

- ② A TWT has beam voltage of 2kV, beam current of 4mA, circuit length of $N=50$, characteristic impedance of helix is 20.2 and is operating at a freq of 8GHz. Determine the gain parameter of the tube and power gain in dB.

Sol Gain parameter (CC) $= \left(\frac{I_0 Z_0}{4V_0} \right)^{1/3} = 21.5 \times 10^{-3}$

$$\text{output power gain} = -9.54 + 47.3 N$$

$$= 41.31 \text{ dB.}$$

③. A TWT operates under the following ^{parameters:-} conditions

Beam voltage : $V_0 = 3 \text{ kV}$

Beam current : $I_0 = 30 \text{ mA}$

characteristic Imp: $Z_0 = 10 \Omega$
of helix

circuit length $N = 50$

freq $f = 10 \text{ GHz}$

Determine a) the gain parameter b) o/p power gain A_p in dB

Sol gain parameter $C = \left(\frac{I_0 Z_0}{4V_0} \right)^{1/3} = \left(\frac{30 \times 10^{-3} \times 10}{4 \times 3 \times 10^3} \right)^{1/3} = 2.92 \times 10^{-2}$

b) ~~from~~ o/p power gain is,

$$A_p = -9.54 + 47.3 \text{ NC} = -9.54 + 47.3 \times 50 \times 2.92 \times 10^{-2}$$

$$= 59.52 \text{ dB}$$

Q4 A ~~conver~~

A TWT has beam voltage of 3 kV, beam current of 30 mA, circuit length of $N = 50$, characteristic impedance of helix is $Z_0 = 10 \Omega$. Determine the gain parameter of the tube and power gain in dB.

Gain parameter $C = \left(\frac{I_0 Z_0}{4V_0} \right)^{1/3} = \left(\frac{30 \times 10^{-3} \times 10}{4 \times 3 \times 10^3} \right)^{1/3} = 2.92 \times 10^{-2}$

output power gain $A_p = -9.54 + 47.3 \text{ NC}$

$= 59.52 \text{ dB}$

PROBLEMS.

(11)

①: A 2-cavity klystron amplifier has the following parameters
 $V_0 = 1000V$, $R_0 = 40k\Omega$, $I_0 = 25mA$, $f = 3GHz$, @ 8
 Gap spacing in either cavity: $d = 1mm$, Spacing b/w the 2 cavities
 $L = 4cm$, effective shunt impedance, excluding beam
 loading: $R_{sb} = 30k\Omega$.

- Find the input gap voltage to give maximum voltage V_2 .
- Find the voltage gain, neglecting the beam voltage loading in the dp cavity.
- Find the efficiency of the amplifier.

Soln: For maximum V_2 , $J_1(x)$ must be maximum. This means $J_1(x) = 0.582$ at $x = 1.841$. The electron velocity just leaving the cathode is,

$$v_0 = (0.593 \times 10^6) \sqrt{V_0} = (0.593 \times 10^6) \sqrt{10^3} = 1.88 \times 10^7 \text{ m/s.}$$

The gap transit angle is

$$\theta_g = \omega \frac{d}{v_0} = 2\pi (3 \times 10^9) \cdot \frac{10^{-3}}{1.88 \times 10^7} = 1 \text{ rad.}$$

The beam coupling coefficient is,

$$B_i = B_o = \frac{\sin \theta_g / 2}{\theta_g / 2} = \frac{\sin (1/2)}{1/2} = 0.952.$$

The dc transit angle b/w the cavities is,

$$\theta_o = \omega T_o = \omega \frac{L}{v_0} = 2\pi (3 \times 10^9) \frac{4 \times 10^{-2}}{1.88 \times 10^7} = 40 \text{ rad.}$$

The maximum input voltage V_1 is then given by,

$$V_{1max} = \frac{2V_0 x}{B_i \theta_o} = \frac{2(10^3)(1.841)}{(0.952)(40)} = 96.5 \text{ V.}$$

② The voltage gain is found as,

$$A_v = \frac{B_o^2 \theta_o}{R_0} \frac{J_1(x)}{x} R_{sb} = \frac{(0.952)^2 (40) (0.582) (30 \times 10^3)}{4 \times 10^4 \times 1.841} = 8.595$$

③. The efficiency can be found by:

$$I_2 = 2 I I_1(x) = 2 \times 25 \times 10^{-3} \times 0.582 = 29.1 \times 10^{-3} \text{ A}$$

$$V_2 = \beta_0 I_2 R_{sb} = (0.952)(29.1 \times 10^{-3})(30 \times 10^3) = 831 \text{ V}$$

$$\text{Efficiency} = \frac{\beta_0 I_2 V_2}{2 I_0 V_0} = \frac{(0.952)(29.1 \times 10^{-3})(831)}{2(25 \times 10^{-3})(10^3)} = \underline{\underline{46.2\%}}$$

④

Q. A two cavity klystron amplifier has the following specifications:

Beam voltage $V_0 = 900 \text{ V}$, Beam current $I_0 = 30 \text{ mA}$, freq $f = 8 \text{ GHz}$,

Gap spacing in either cavity: $d = 1 \text{ mm}$ cavity, Spacing b/w

centres of cavities $L = 4 \text{ cm}$, effective shunt resistance $R_{sh} = 49 \text{ G}\Omega$.

Determine a) The electron velocity (b) The dc transit time of electron (c) The input voltage for maximum output voltage. d) The voltage gain in dB.

Sol a) electron velocity $v_0 = 0.593 \times 10^6 \sqrt{V_0}$
 $= 0.593 \times 10^6 \sqrt{900 \text{ V}} = 17.79 \times 10^6 \text{ m/sec}$

(b) dc transit time of electrons :-

$$\theta_0 = \omega T_0 = \omega \frac{L}{v_0}$$

$$T_0 = \frac{L}{v_0} = \frac{4 \times 10^{-2}}{17.79 \times 10^6} = 0.225 \times 10^{-8} \text{ sec.}$$

(c) maximum i/p voltage $V_1(\text{max}) = \frac{V_0 \times 3.68}{\beta_1 \theta_0}$

$$\theta_0 = \omega T_0 = 2\pi \times 8 \times 10^9 \times 0.225 \times 10^{-8} = 113.1 \text{ rad}$$

$$\beta_1 = \sin \theta_g / 2$$

$$\theta_g = \omega \frac{d}{v_0} = \frac{2\pi \times 8 \times 10^9 \times 10^{-3}}{17.79 \times 10^6} = 2.835 \text{ rad}$$

$$\frac{\theta_g}{2} = 1.413 \text{ rad}, \sin \theta_g / 2 = 0.988$$

$$\therefore \beta_0 = \beta_1 = \frac{\sin(\theta_g/2)}{\theta_g/2} = \frac{0.988}{1.413} = 0.699$$

$$V_1(\max) = \frac{900 \times 3.68}{0.699 \times 113.1} = 41.894 \text{ V} \quad (10)$$

(11) voltage gain ..

$$A_v = \frac{V_2}{V_1} = \frac{B_0^2 \sin^2 \theta_1(x)}{R_0} R_{sh}.$$

$$= \frac{(0.699)^2 \times (113.1) (0.582) 40 \times 10^3}{30 \times 10^3 \times 7.844} = \underline{\underline{23.293}}$$

(3) A reflex klystron operates at the peak mode of $n=2$ with beam voltage $V_0 = 300 \text{ V}$. Beam current $I_0 = 20 \text{ mA}$, signal voltage $V_1 = 40 \text{ V}$. Determine (i) the input power in watts. (ii) output power in watts (iii) efficiency.

Sol (i). $P_{ac} (\text{Input power}) = V_0 I_0 = 300 \times 20 \times 10^{-3} = 6 \text{ W}.$

(ii) $P_{ac} (\text{Output "}) = \frac{2 V_0 I_0 \times J_1(x')}{2n\pi - \pi/2} = \frac{2 \times 6 \times 1.25}{(2 \times 2 \times \pi) - \pi/2} = 1.36 \text{ watts}.$

(iii) $\eta = \frac{P_{ac}}{P_{dc}} \times 100 \% = \frac{1.36}{6} = 22.7 \%$

(4) A Reflex klystron operates at the peak mode of $n=2$ with $V_0 = 280 \text{ V}$, $I_0 = 22 \text{ mA}$ and signal voltage $V_1 = 30 \text{ V}$. Determine a) the input power b) o/p power c) the η .

Sol a) $P_{ac} (\text{i/p power}) = V_0 I_0 = 280 \times 22 \times 10^{-3} = 6.16 \text{ watts}$

b) $P_{ac} (\text{o/p "}) = \frac{2 V_0 I_0 \times J_1(x')}{2n\pi - \pi/2} = \frac{2 \times 6.16 \times 1.25}{2 \times 2 \times \pi - \pi/2} = 1.4 \text{ watt}$

c) $\eta = \frac{P_{ac}}{P_{dc}} \times 100 = \frac{1.40}{6.16} \times 100 = 22.74 \%$

(3) A Reflex Klystron operates under the following conditions: (mks system)

$V_0 = 600 \text{ V}$, $L = 1 \text{ mm}$, $R_{sh} = 15 \text{ k}\Omega$, $\frac{e}{m} = 1.759 \times 10^{11} \text{ C/kg}$, $f_r = 9 \text{ GHz}$
 the tube is oscillating at first the peak of the $n=2$ mode or $1\frac{3}{4}$ mode. Assume that the transit time through the gap and beam loading can be neglected.

- a) Find the value of the repeller voltage V_r .
 b) Find the direct current necessary to give a microwave gap voltage of 200 V . c) What is the efficiency of undulator condenser.

$$\text{sol} \quad \frac{V_0}{(V_r + V_0)^2} = \left(\frac{e}{m} \right) \frac{(2\pi n - \pi/2)^2}{8\omega^2 L^2}$$

$$= (1.759 \times 10^{11}) \cdot \frac{(2\pi n - \pi/2)^2}{8(2\pi \times 9 \times 10^9)^2 (10^{-3})^2} = 0.832 \times 10^{-3}$$

$$(V_r + V_0)^2 = \frac{600}{0.832 \times 10^{-3}} = 0.721 \times 10^6$$

$$\underline{V_r = 250 \text{ V}}$$

(b) Assume that $B_0 = 1$ since $V_2 = \frac{I_2 R_{sh}}{2I_0 J_1(x')}$
 $= 2I_0 J_1(x') R_{sh}$

The direct current I_0 is

$$I_0 = \frac{V_2}{2J_1(x') R_{sh}} = \frac{200}{2(0.582)(15 \times 10^3)}$$

$$= \underline{11.45 \text{ mA}}$$

(c) Efficiency is:-

$$\eta = \frac{2x' J_1(x')}{2\pi n - \pi/2} = \frac{2(1.841)(0.582)}{2\pi(2) - \pi/2}$$

$$= \underline{19.49 \%}$$

MICROWAVE SOLID STATE DEVICES7.1 INTRODUCTION.UNIT - IV (GUNN DIODE)

In the recent past there has taken place a tremendous research activity for development of better low noise, high frequency, greater bandwidth, lesser switching time and other improvements in the performance characteristics for achieving the applications such as amplification, oscillation, switching, limiting, frequency multiplication. The microwave tubes were used for the above applications by employing velocity modulation. In this endeavor, several semiconductor microwave devices have been developed which include three terminal devices such as bipolar and field effect transistors and two terminal devices such as transferred electron devices (GUNN DIODES), avalanche transit time devices (IMPATT, TRAPATT, BARATT, parametric devices), tunnel diodes, varactors etc.

7.2 - CLASSIFICATION OF SOLID STATE MICROWAVE DEVICES

Solid state microwave devices can also be classified

1. Based on their electrical behaviour
2. Based on their construction

Based on electrical behaviour

- a) Non-linear resistance type: eg varistors (variable resistances)
- b) Non-linear reactance type: eg varactors (variable reactors)
- c) negative resistance type: eg. Tunnel diode, IMPATT, GUNN DIODE
- d) controllable impedance type: eg. PIN diode.

Based on construction

- a) Point ~~to~~ contact diodes
- b) Schottky barrier diodes

c) metal oxide semiconductor devices (mos)

d) metal insulation devices.

7.3. APPLICATIONS.

The solid state devices have many applications, viz amplification, detection, down conversion, up conversion, modulation, switching, limiting, power generation, phase shifting etc. These devices have been commonly employed for several applications that include modern communication systems, radars, navigation, medical and biological equipment and other industrial electronic products.

7.4. TRANSFERRED ELECTRON DEVICES (TED'S).

7.4.1 INTRODUCTION.

TED's are bulk devices having no junction, or gates as compared to a microwave transistors which operate with either junction or gates. TED's are fabricated from compound semiconductors such as GaAs, InP or GaTe as against Ge and Si of transistors. TED's operate with hot electrons whose energy is very much greater than the thermal energy. Gunn Diode is an example of this kind.

The common characteristics of all active two terminal device (solid state devices) is their negative resistance. In positive resistance $\rightarrow$ the current through resistance and voltage across it are in phase. the voltage drop across positive resistance is positive and a power of I^2R is dissipated in the resistor. In negative resistance the current and voltage are out of phase by 180° , the voltage drop across it is negative and a power of $(-I^2R)$ is generated by the power supply associated with the negative resistance. In other words positive resistances absorb power and negative resistance generate power (active devices).

7.4.2 GUNN DIODE - PRINCIPLE

GUNN DIODES are named after J.B. GUNN (1963), who discovered periodic fluctuations of current passing through the n-type GaAs specimen when the applied voltage exceeded a certain critical value. Gunn effect can be explained on the basis of two valley theory or Ridley Watkins - Hilsum (RWH) theory or the transferred electron mechanism.

Basic principle or mechanism involved in the operation of bulk n type GaAs devices is the transfer of electrons from lower conduction valley the L-valley to upper subsidiary valley the U-valley.

7.4.3

RWH THEORY.

As shown from Fig 7.1, the curvature of the two valleys in the conduction band, also called the subbands are different, so that an electron in L-valley has a smaller effective mass ($m_1 = 0.072 m_0$) than one in the U-valley ($m_2 = 1.2 m_0$). The effective masses mean different mobilities for the L-valley ($\mu_1 = 0.5 \text{ m}^2/\text{volt sec}$) and the U-valley ($\mu_2 = 0.01 \text{ m}^2/\text{volt sec}$) respectively. The ratio of density of states in the U-valley to that in the L-valley is about 60. Thus the upper valley has a very high density of states compared with the $k=0$ location.

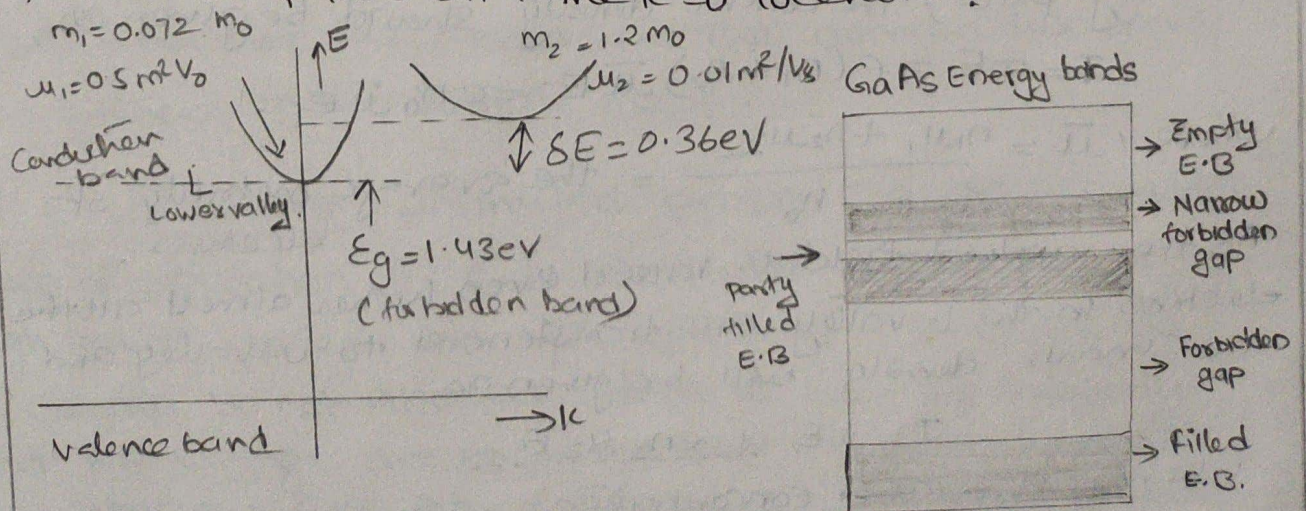


Fig. 7.1

At low (or zero) electric fields, conduction electrons are distributed in a manner determined by energy separation ΔE , the lattice temperature T_0 and the density of states. With the typical values stated in the figure, most of the electrons at low electric fields and at low lattice temperature will occupy states in the L-valley and carry ohmic current $J = \sigma E$.

with $\sigma \approx p n_1 \mu_1 = e n_0 \mu_1$, where n_1 is the carrier concentration in the L-valley and is assumed to be equal to the total carrier concentration ~~and the carrier concentration is n_0~~ n_0 and μ_1 is the mobility in the L-valley.

As the applied field is increased, the electrons gain energy from it and move upward in the U-valley. Actually the probability of this inter valley transfer of electrons is good as there are many available states in the U-valley. As the electrons transfer to this U-valley, their mobility decreases and the effective mass is increased thus decreasing the carrier density J and hence the negative differential conductivity. There is a certain threshold field, approximately 33 kV/cm above which this inter valley transfer (also known as the population inversion) & charges from lower L-valley to U-valley or the transfer electron effect takes place. As the transfer of electrons is taking place, the current density should be given as,

$$J = \sigma E = e(n_1 + n_2) \bar{\mu} E = e n_0 \bar{\mu} E.$$

where, $\bar{\mu} = \frac{n_1 \mu_1 + n_2 \mu_2}{n_0}$ = the average mobility of electrons.

As the applied field is raised even higher almost all the electrons in the L-valley are transferred to U-valley and the current density will be given as,

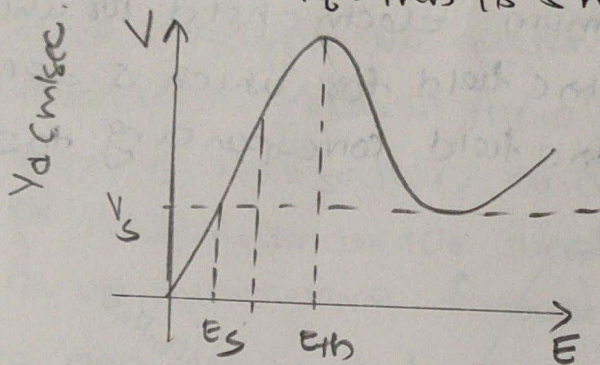
$$J = \sigma E \approx e n_2 \mu_2 E$$

where, n_2 = carrier concentration

μ_2 = mobility in U-valley

1. Transit Time Domain mode ($fL \approx 10^7$ cm/sec).

This is also called the Gunn mode. Here $fL = 10^7$ cm/s $= v_t$, when $v_t = v_s$ the sustaining velocity, the high field domain is stable. In that case the oscillation period $=$ transit time i.e. $\tau_0 = \tau_t$. This is shown in Fig 7.3.



2. Delayed Domain mode (10^6 cm/sec $< fL < 10^7$ cm/sec).

When the transit time is chosen so that the domain is collected while $E < E_{th}$ a new domain cannot form until the field rises again above threshold. Here oscillation period is greater than transit time $\tau_0 > \tau_t$.

3. Quenched domain mode ($fL > 2 \times 10^7$ cm/sec).

If the bias field drops below sustaining field E_s during the negative half cycle the domain collapses before it reaches the anode. i.e. the domain disappears somewhere in the sample itself.

4. LSA MODE - limited space charge accumulation mode of operation for Gunn oscillator on this mode gives high power output with high efficiency. In this mode, the domain is not allowed to form at all. The frequency and RF voltage are so chosen that the domains do not have sufficient time to form while the field is above threshold. As a result most of the domains are maintained in the negative conductance state during a large part of the ^{RF} cycle.

7.4.4 CHARACTERISTICS

Thus a J_V vs E curve is obtained similar to V - I characteristics of $p-n$ junction diodes for the voltage controlled bulk negative conductance GaAs sample as in Fig 7.2

where J_m = maximum current density

J_v = valley current density

E_m = Maximum electric field reqd

E_a = Maximum electric field for which $J = \sigma E$

E_b = Electric field for which $S = \epsilon_0 \mu_0 E$ holds

E_v = electric field corresponding to J_v .

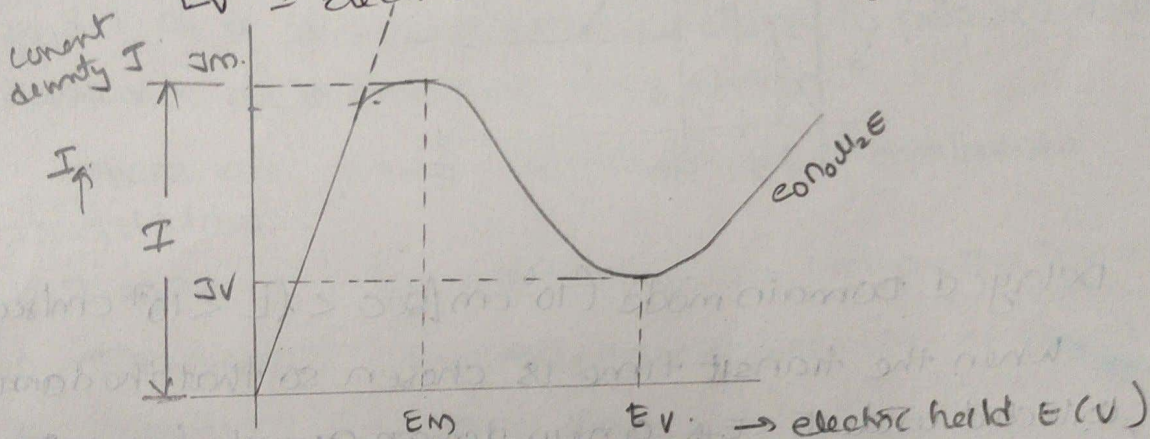


Fig 7.2 IE characteristics of a Gunn diode.

Hence these JE characteristics represent the V-I characteristics of GUNN DIODE.

The region of the characteristic between E_m and E_0 where current density decreases with increasing electric field is one of negative differential resistivity (NDR).

7.4.5. MODES OF OPERATION :-

depending on the material parameters and operating conditions. A Gunn effect oscillator can be made to oscillate in any of the 4 frequency modes.

1. Domain mode (dipole transit time).
2. Delayed mode
3. Quenched domain mode
4. Limited space charge accumulation (LSA) mode.

7.6.7.5. AVALANCHE DEVICES.

INTRODUCTION:

It is possible to make a microwave diode exhibit negative resistance by having a delay between voltage and current in an avalanche together with transit time through the material. Such devices are called Avalanche transit time devices. They use carrier impact ionisation and drift in the high field region of a semiconductor junction to produce negative resistance at microwave frequencies.

There are three distinct modes of avalanche oscillators:-

1. IMPATT :- Impact Ionisation Avalanche Transit Time device
2. TRAPATT :- Trapped plasma Avalanche Triggered Transit device
3. BARITT :- Barrel Injected Transit Time device

— X — .

UNIT - VIII

MICROWAVE MEASUREMENTS.

8.1 INTRODUCTION:-

Microwave measurements are concerned with distributed circuit elements, which are acted upon by electromagnetic waves having interdependent E and H fields. The time varying magnetic fields generate the time varying electric fields and viceversa.

microwave system which include power generation, transmission and detection are in many ways different than low frequency radio wave system. Hence the measurement techniques and parameters assumed are also different.

At microwave frequencies, most important parameters to be measured are the power, impedance, frequency, attenuation, VSWR, Phase shift, Noise factor etc. we shall concentrate on some of the above mentioned parameters.

8.2 Description of BENCH, DIFFERENT BLOCKS & THEIR FEATURES.

The general set up for measurement of any parameter in microwave is normally done by a microwave bench. Such a set up is shown in Fig 8.1

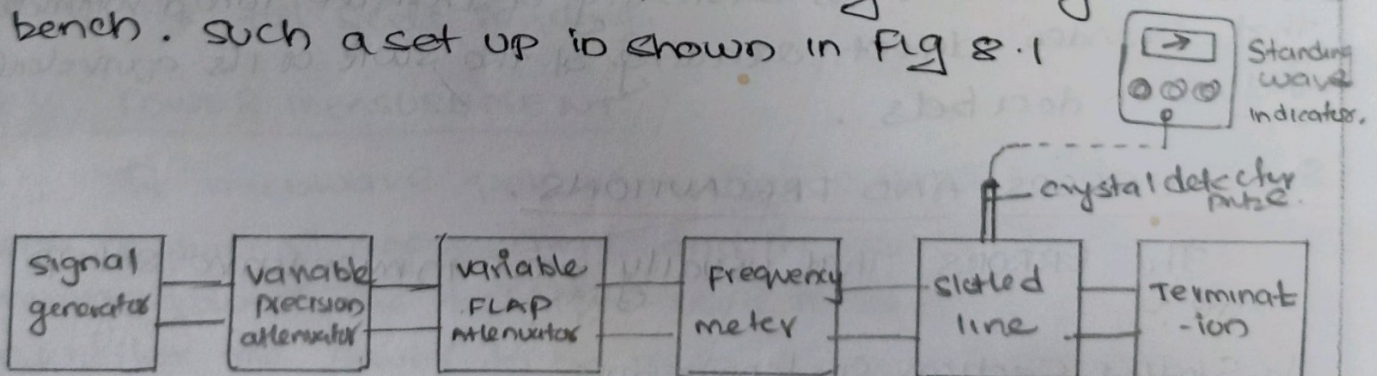


Fig 8.1 microwave Bench set-up.

Signal generator - Is a microwave source whose output is of the order of milliwatts. It could be a Gunn Diode oscillator or a backward wave oscillator or a klystron tube. It can provide either a continuous wave (CW) or square wave modulated at an audio rate which is normally 1 kHz.

VARIABLE PRECISION ATTENUATOR - It can provide 0 to 50 dB attenuation above its insertion loss.

Variable Flat Attenuator :- It is also used in addition, whose calibration can be checked against readings of the precision attenuator.

Frequency meter :- is used for direct reading of frequency that consists of single cylindrical cavity which can be adjusted to resonance and is slot coupled to the waveguide.

Slotted line - It is basically used for measuring the standing wave ratio. A coupling probe moving along the waveguide can be used to detect the standing wave pattern present inside the waveguide.

CRYSTAL DETECTOR - Inserted in the E-probe of the slotted line is contained in the crystal detector mount at the end of the waveguide run is used to detect the modulated signal.

SWR & indicator is basically a sensitive tuned voltmeter that provides direct reading of the SWR or its equivalent value in decibels.

8.3. ERRORS AND PRECAUTIONS.

The ERRORS that usually occur in microwave setup are:-

1. Slotted Section System Errors

- Error due to frequency agility
- Coupling error
- Standing wave detector error.

2) Square Law detector error

microwave crystals are usually made of germanium or silicon crystal, ~~they are~~ their characteristics are strongly dependent upon the contact between the wire and semiconductor and is subjected to defects such as uncertain response law, etc.

3) Probe error — The presence of probe introduces a field distortion which effects the pattern of the standing wave. the distortion is more pronounced at a voltage maximum.

4) Slot error — The slot in the slotted section leads the waveguide and produces slight shift in its waveguide wave length; and the end of the slot introduces a discontinuity effect.

PRECAUTIONS

1. The amplifier should be run at almost full gain.
2. The probe depth should be small and fixed.
3. The tuning stub should be adjusted for maximum power transfer.
4. The noise level of the amplifier when the source is not oscillating, should not be more than a very small fraction of the full scale reading.

8.4. POWER MEASUREMENT

The microwave power inside a waveguide is invariant with position of measurement and the power measured is the average power. The technique used to measure power depends on whether the power to be measured is low or high. Thus the measurement of power is divided into 3 categories:-

1. Measurement of low power ($0.01\text{mW} - 10\text{mW}$) (Bolometer technique)
2. Measurement of medium ^{microwave} power ($10\text{mW} - 1\text{W}$) (calorimetric technique).

c) measurement of high microwave power ($>10\text{W}$) - calorimetric technique

8.41 Measurement of low microwave power.

For measuring low microwave powers, devices such as bolometers and thermocouples whose resistance changes with the applied power. Bolometer is a simple temperature device whose resistance with temperature. These are of two types :- Bolometers and Thermistors.

Bolometers have positive temperature coefficient and their resistance increases with an increase in temperature as shown in Fig 8.2 (a) and Thermistors have negative temperature coefficient of resistance and their resistance decreases with increase in temperature as shown in Fig 8.2 (b).

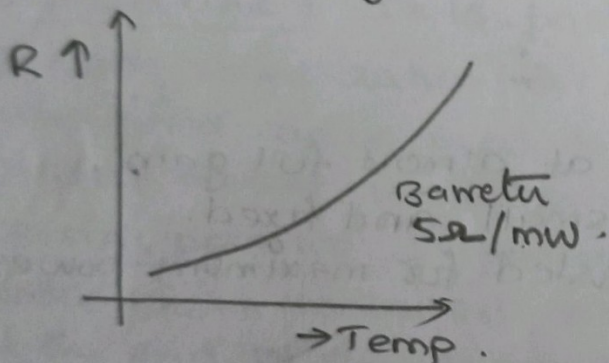


Fig 8.2(a)

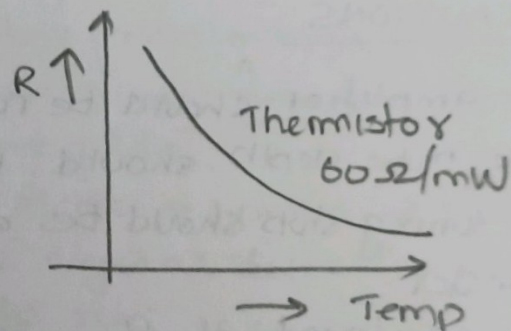


Fig 8.2(b)

A bolometer such as crystal diode is a square law device and it produces a current that is proportional to the applied power i.e. square of the applied voltage, rather than the applied voltage.

Bolometer is mounted inside the waveguide as shown in Fig 8.3, where the bolometer itself is used as a load, with the operation resistance as R_L . Now the low microwave power which is to be measured is applied. Some power is absorbed

in the bolometer load and dissipated as heat and the resistance changes to R_2 . This change in resistance ($R_1 - R_2$) is proportional to the microwave power which can be measured using a bridge.

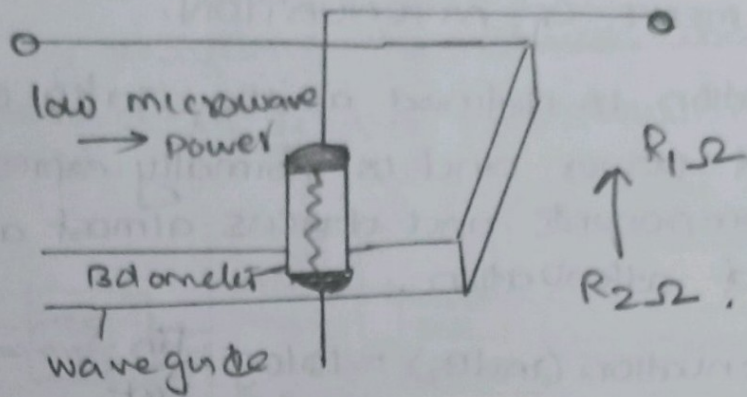


Fig 8.3.

In the balanced bolometer bridge technique, the bolometer itself is made to be one of the arms of the bridge as shown in Fig 8.4.

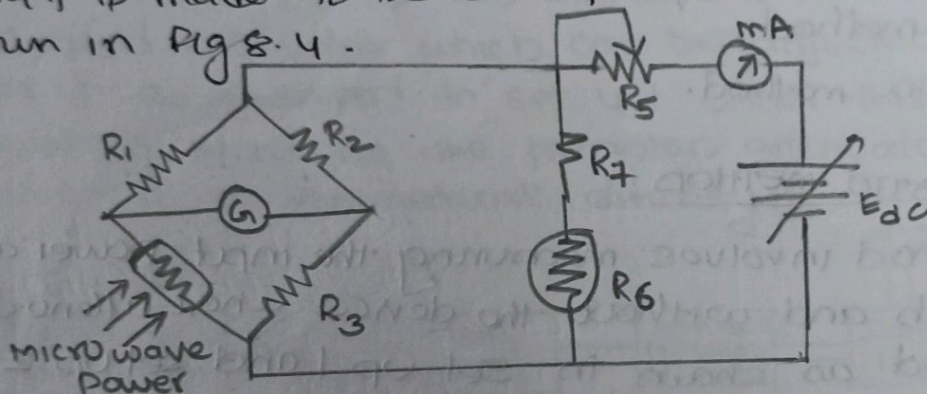


Fig 8.4

Initially the bridge is balanced by adjusting R_5 , which varies the dc power applied to the bridge and the bolometer element is brought to a predetermined operating resistance before microwave power is applied. Let the voltage of the battery be E , at balance. The microwave power is now applied and this power gets dissipated in the bolometer. The bolometer heats up and it changes its resistance. Therefore the bridge becomes unbalanced. The applied dc power is changed to E_2 to get back the balance and this change in dc battery voltage ($E_1 \sim E_2$) will be proportional to the microwave

power. Alternately the detector G can be directly calibrated in terms of microwave power so that when the bridge is unbalanced, the detector reads the microwave power directly.

8.5 MEASUREMENT OF ATTENUATION.

Attenuation is defined as the ratio of input power to the output power and is normally expressed in decibels. Microwave components and devices almost always provide some degree of attenuation.

$$\text{i.e. Attenuation (indBs)} = 10 \log \frac{P_{in}}{P_{out}} \quad \text{--- (1)}$$

where P_{in} = input power and P_{out} = output power.

Attenuation can be measured by two methods:-

- Power ratio method
- RF substitution method.

8.5.1. POWER RATIO METHOD

This method involves measuring the input power and output power with and without the device whose attenuation is to be measured as shown in set up 1 and set up 2 of Fig 8.5(a) and 8.5(b). The powers are measured in each set up as P_1 and P_2 . The ratio of power P_1/P_2 expressed in decibels gives the attenuation as in eq (1).

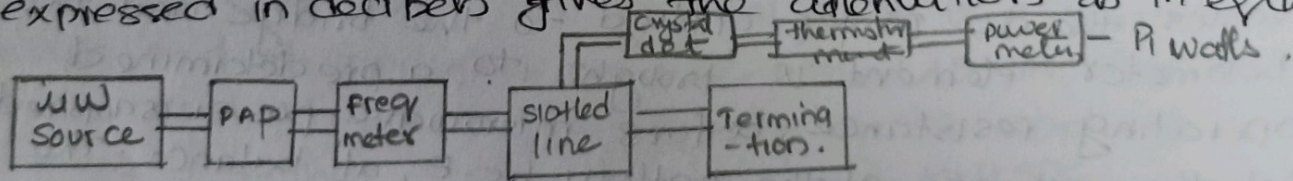


Fig 8.5(a)

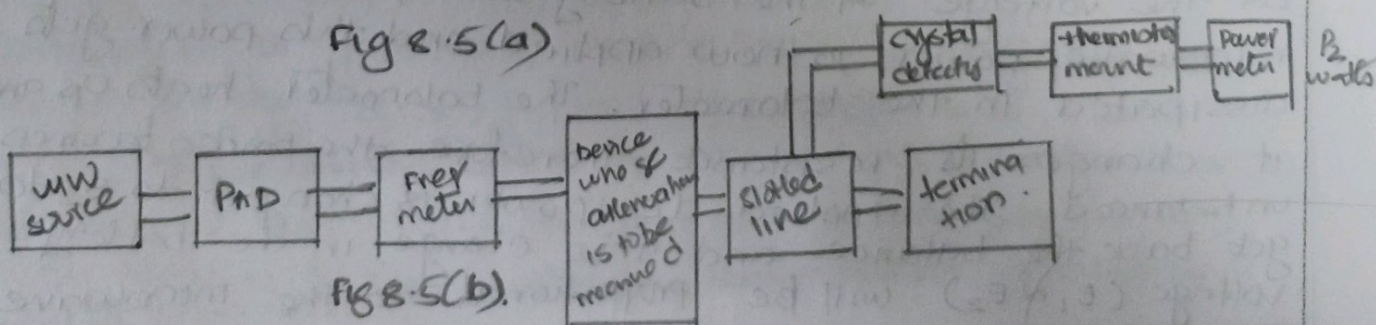


Fig 8.5(b)

85.2. RF substitution method.

This method overcomes the drawback of power ratio method since here we measure attenuation at a single power position. The method consists of measuring the d.p. power say P' by including the network whose attenuation is to be measured in set up 1 as shown in Fig 8.6(a).

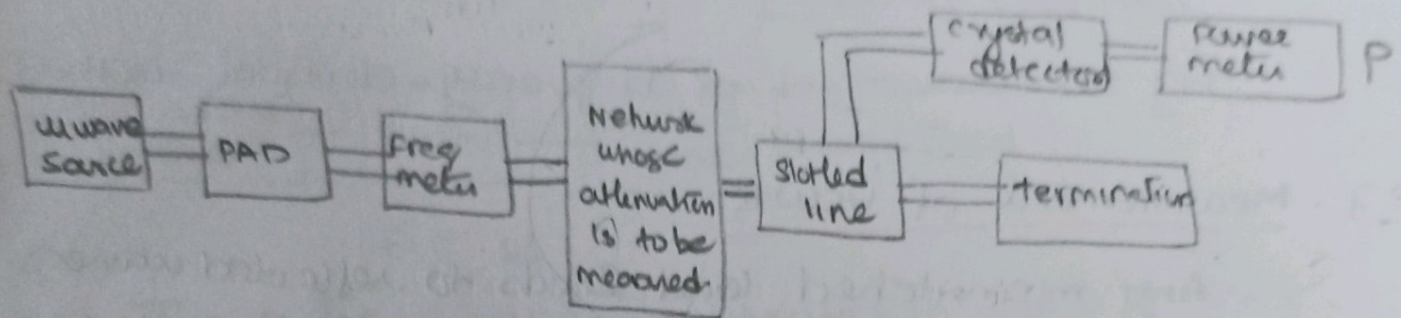


Fig 8.6(a).

In set up 2 (Fig 8.6(b)) this network is placed by a precision calibrated attenuator which can be adjusted to obtain the same power P' as measured in set up 1. Under this condition the attenuation read on the precision attenuator would give attenuation of the network directly.

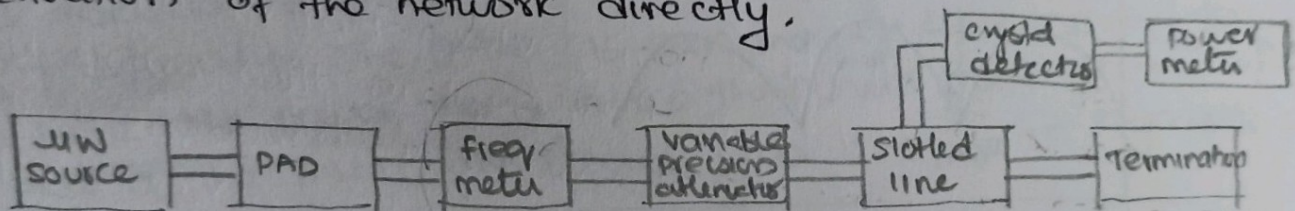


Fig 8.6(b)

8.6 - Measurement of FREQUENCY.

Microwave frequency can be measured by either electronic or mechanical techniques.

86.1 ELECTRONIC Technique

These techniques generally are more accurate but expensive. Frequency counters or high frequency heterodyne systems can be used. Here the unknown frequency is compared with harmonics of a known lower frequency. It is compared with harmonics of a known lower frequency by use of a low

frequency generator, a harmonic generator and a mixer as shown in Fig 8.7.

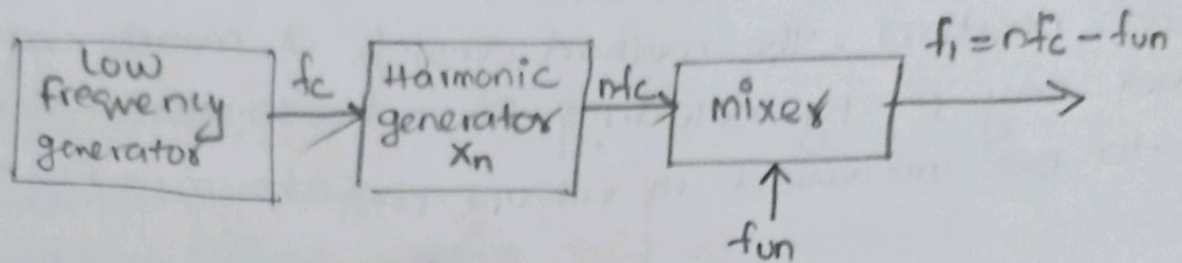


Fig 8.7 Measurement of frequency-electronic method.

8.7. Measurement of VSWR,

Any mismatched load leads to reflected waves resulting in standing waves along the length of the line. The ratio of maximum to minimum voltage gives the VSWR as shown in Fig 8.8.

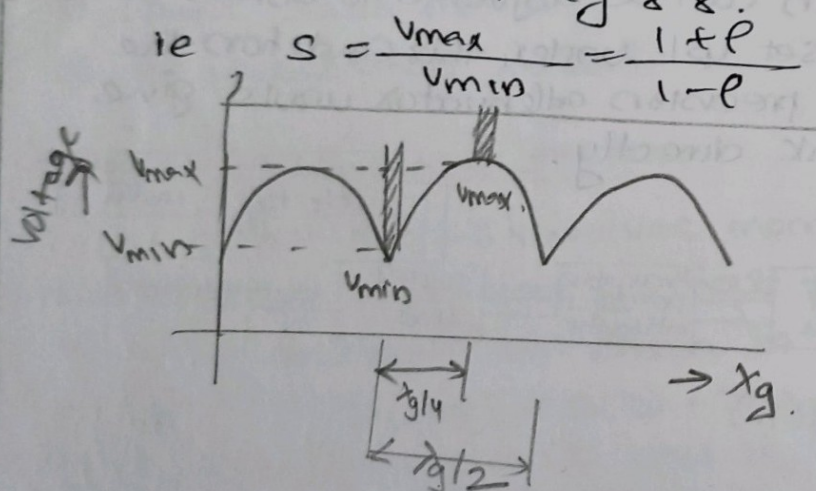


Fig: 8.8.

8.7.1. Measurement of Low VSWR's ($S < 10$)

Values of VSWR not exceeding 10 are very easily measured with the set up shown in Fig 8.8.1 and can be read off directly on the VSWR meter calibrated as shown in Fig 8.8.1. The measurement basically consists of simply adjusting the attenuator to give an adequate reading on the meter, which is a D.C. millivolt meter. The probe

on the slotted waveguide is moved to get maximum reading on the meter (corresponding to V_{max}). The attenuation is now adjusted to get full scale reading. The full scale reading is noted down. Next the probe on the slotted line is adjusted to get minimum reading on the meter (corresponding to V_{min}). The ratio of first reading to the second (i.e. V_{max}/V_{min}) gives the VSWR.

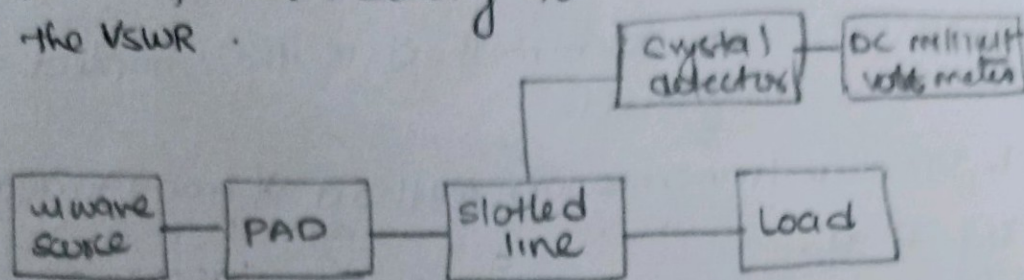


Fig. 8.8.1.

The meter itself can be calibrated in terms of VSWR.

8.7.2. Measurement of high VSWR (>10).

For VSWR's > 10, we use what has come to be known as the double minimum method. In this method, the probe is inserted to a depth where the minimum can be read with out difficulty. The probe is then moved to a point where the power is twice the power point on the other side of the minimum (say d_2). as shown in fig 8.8.2, we get,

$$2 P_{min} \propto \frac{V_a^2}{2}$$

$$\frac{1}{2} = \frac{V_{min}^2}{V_a^2}$$

$$\text{or } V_a^2 = 2(V_{min})^2$$

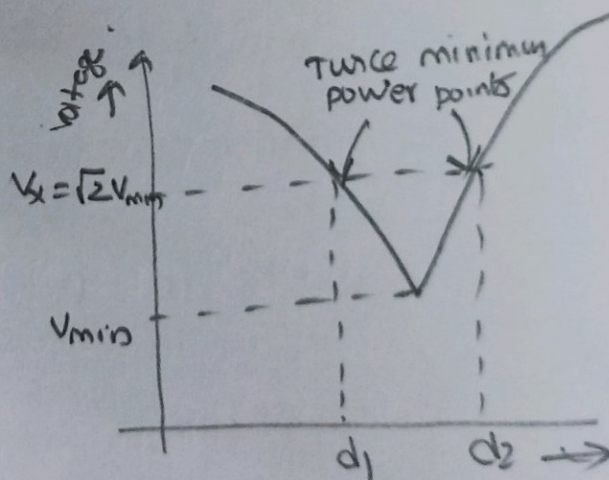
$$\text{or } V_a = \sqrt{2} V_{min}$$

Further for TE_{10} mode, $\lambda_c = 2a$,

$$\lambda_0 = c/f$$

$$\beta = \frac{2\pi}{\lambda_g} = \frac{2\pi}{\lambda_0 \sqrt{1 - (\lambda_0/\lambda_c)^2}}$$

Then VSWR can be calculated using the distance (cm) formula, $VSWR = \lambda_g / \pi (d_2 - d_1)$.



8.8 MEASUREMENT OF Q

There are several methods for measuring the Q of a cavity resonator.

1. Transmission method
2. Impedance method
3. Transient delay & decrement method.

Among these transmission method is the simplest and is described here in detail:-

The set up for transmission method of measuring Q is shown in fig 8.9.

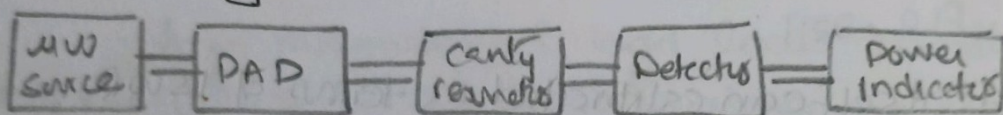


Fig 8.9, set up for measurement of a cavity resonator using transmission method.

In this method, the cavity resonator is used as a transmission device and the output signal is measured as a function of the frequency resulting in the resonance curve.

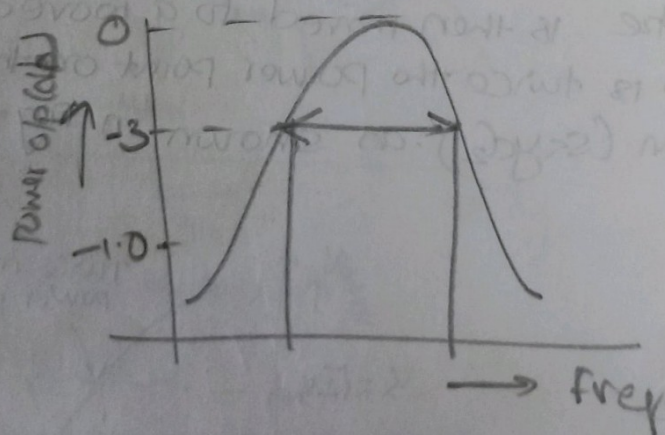
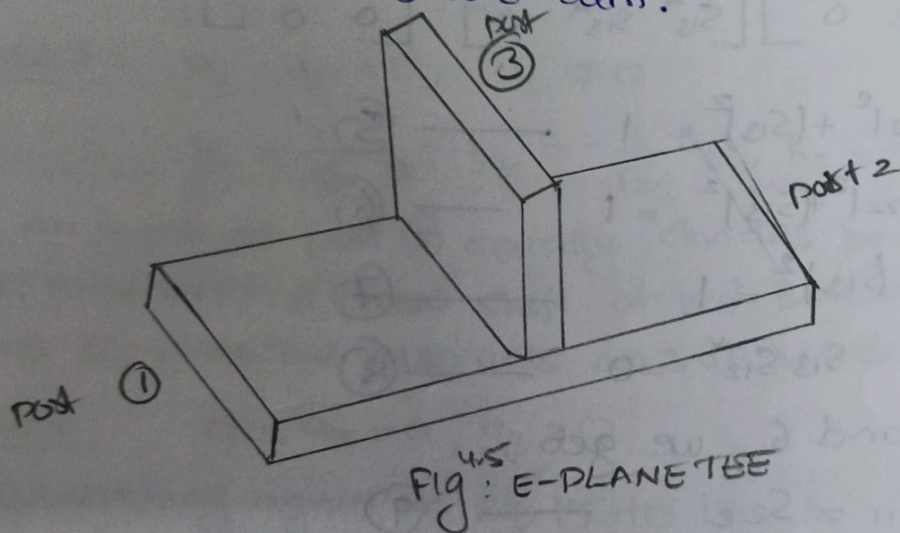


Fig 8.10 Resonance curve.

4.2 S-MATRIX FOR E-PLANETEE.

The scattering matrix of an E-plane Tee can be used to describe its properties. In general the power out of port ③ (side or E arm) is proportional to the difference between the instantaneous powers entering from ports ① and ② (Fig.

Also the effective value of the power leaving the E-arm is proportional to the phase difference between the power entering ports ① and ②. When powers entering the main arms (ports ① and ②) are in phase opposition, maximum power comes out of port 3 or E-arm.



Since it is a three port junction the scattering matrix can be derived as follows:

1. $[S]$ is a 3×3 matrix since there are 3 ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

2. The scattering coefficient $S_{23} = -S_{13}$ — (1)

Since outputs at ports (1) and (3) are out of phase by 180° with an input at port (2).

3. If port (3) is perfectly matched to the junction,

$$S_{33} = 0 \quad \text{--- (2)}$$

4. From symmetric property $S_{ij} = S_{ji}$

$$S_{12} = S_{21}, S_{13} = S_{31}, S_{23} = S_{32} \quad \text{--- (3)}$$

with the above properties of (2) and (3), $[S]$ becomes,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \quad \text{--- (4)}$$

5. From unitary property $[S] = [S]^* = [I]$.

$$\text{i.e.} \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 C_1: |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{--- (5)}$$

$$R_2 C_2: |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{--- (6)}$$

$$R_3 C_3: |S_{13}|^2 + |S_{13}|^2 = 1 \quad \text{--- (7)}$$

$$R_3 C_1: S_{13} \cdot S_{11}^* - S_{13} S_{12}^* = 0 \quad \text{--- (8)}$$

equating eq (5) and 6, we get,

$$S_{11} = S_{22} \quad \text{--- (9)}$$

$$\text{From eq (7), } S_{13} = \frac{1}{\sqrt{2}} \quad \text{--- (10)}$$

$$\text{From eq (8), } S_{13}(S_{11}^* - S_{12}^*) = 0 \text{ or } S_{11} = S_{12} = S_{22} \quad \text{--- (11)}$$

using these values (eq 9 to 11) on eq (5),

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$$

$$\text{or } 2|S_{11}|^2 = \frac{1}{2} \text{ or } S_{11} = \frac{1}{2} \quad \text{--- (12)}$$

substituting the values from 10 to 11, the $[S]$ matrix of eq 4 becomes:

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \quad \text{--- (13)}$$

we know, from eq 8-matrix $[b] = [S][a]$.

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \text{--- (14)}$$

$$b_1 = \frac{1}{2} a_1 + \frac{1}{2} a_2 + \frac{1}{\sqrt{2}} a_3 \quad \text{--- (15)}$$

$$b_2 = \frac{1}{2} a_1 + \frac{1}{2} a_2 - \frac{1}{\sqrt{2}} a_3 \quad \text{--- (16)}$$

$$b_3 = \frac{1}{\sqrt{2}} a_1 - \frac{1}{\sqrt{2}} a_2 \quad \text{--- (17)}$$

Case 1: $a_1 = a_2 = 0, a_3 \neq 0$

$$b_1 = \frac{1}{\sqrt{2}} a_3; b_2 = -\frac{1}{\sqrt{2}} a_3; b_3 = 0.$$

ie an input at port (3) equally divides between (1) and (2) but introduces a phase shift of 180° between the two outputs. Hence E-plane Tee also acts as 3 dB splitter.

Case 2: $a_1 = a_2 = a, a_3 = 0$

Substituting again in eq 15 to 17, $b_1 = \frac{a}{2} + \frac{a}{2}$;

ie equal inputs at port (1) and port (2) result $b_2 = \frac{a}{2} + \frac{a}{2}$

in no output at port (3).

$$b_3 = \frac{1}{\sqrt{2}} a - \frac{1}{\sqrt{2}} a = 0$$

Case 3: $a_1 \neq 0, a_2 = 0, a_3 = 0$

$$\text{Hence, } b_1 = \frac{a_1}{2}; b_2 = \frac{a_1}{2}; b_3 = \frac{a_1}{\sqrt{2}}.$$

Similarly, all combinations of inputs and outputs can be defined.

4.4.3. S-matrix for H-PLANE TEE

The properties of an H-plane TEE can be defined by its $[S]$ matrix. The order of scattering matrix is 3×3 since there are 3 possible inputs and 3 possible outputs.

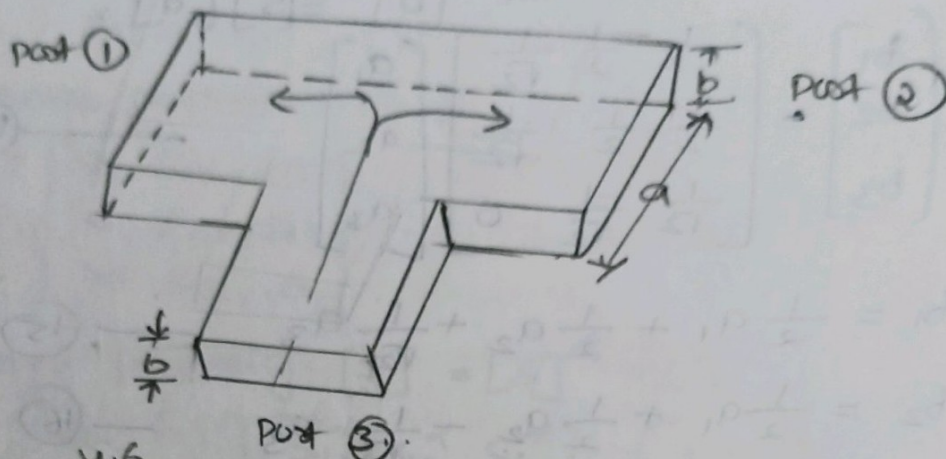


Fig. 4.6 H-plane TEE

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \text{--- (1)}$$

Now, we determine the S-parameters by applying the properties of $[S]$.

1. Because of plane of symmetry of the junction scattering coefficients S_{13} and S_{23} must be equal.

$$\therefore S_{13} = S_{23}$$

2. From the symmetric property, $S_{ij} = S_{ji}$

$$\therefore S_{12} = S_{21}; S_{23} = S_{32} \stackrel{1}{=} S_{13}, S_{13} = S_{31}$$

3. Since port 3 is perfectly matched to the junction

$$S_{33} = 0$$

With these properties $[S]$ matrix of eq (1) becomes,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \quad \text{--- (2)}$$

i.e., we have four unknowns.

4. From the unitary property,

$$[S][S]^* = [I]$$

$$\text{i.e., } \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{21}^* & S_{22}^* & S_{23}^* \\ S_{31}^* & S_{32}^* & S_{33}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

multiplying we get,

$$R_1 C_1 : |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{--- (5)}$$

$$R_2 C_2 : |S_{12}|^2 + |S_{22}|^2 + |S_{23}|^2 = 1 \quad \text{--- (6)}$$

$$R_3 C_3 : |S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1 \quad \text{--- (7)}$$

$$R_3 C_1 : S_{13} S_{11}^* + S_{23} S_{12}^* + S_{33} S_{31}^* = 0 \quad \text{--- (8)}$$

From eq. 3 and 4, we get

$$|S_{11}|^2 = |S_{22}|^2 \quad \text{--- (9)}$$

$$\therefore S_{11} = S_{22}$$

$$\text{From eq. (5), we } 2|S_{13}|^2 = 1, \text{ or } S_{13} = \frac{1}{\sqrt{2}}$$

$$\text{From eq. (8), } S_{13} \neq 0, S_{11}^* + S_{12}^* = 0, \text{ or } S_{11}^* = -S_{12}^* \\ \text{or } S_{11} = -S_{12} \text{ or } S_{12} = -S_{11} \quad \text{--- (10)}$$

using these equations, in eq. (3) we get,

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1 \text{ or } 2|S_{11}|^2 = \frac{1}{2} \text{ or } S_{11} = \frac{1}{2} \quad \text{--- (11)}$$

$$\therefore \text{from eq. 8 and 9, } S_{12} = -\frac{1}{2} \quad \text{--- (12)}, S_{22} = \frac{1}{2} \quad \text{--- (13)}$$

substituting for S_{13} , S_{11} , S_{12} and S_{22} from eq. 7 to 11, in eq. 2, we get,

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \quad \text{--- (14)}$$

we know that $[b] = [S][a]$ hence

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\text{ie, } b_1 = \frac{1}{2} a_1 - \frac{1}{2} a_2 + \frac{1}{\sqrt{2}} a_3 \quad \text{--- (13)}$$

$$b_2 = -\frac{1}{2} a_1 + \frac{1}{2} a_2 + \frac{1}{\sqrt{2}} a_3 \quad \text{--- (14)}$$

$$b_3 = \frac{1}{\sqrt{2}} a_1 + \frac{1}{\sqrt{2}} a_2 \quad \text{--- (15)}$$

Case 1: $a_3 \neq 0, a_1 = a_2 = 0$

ie input is given at port ③ and no inputs at port ① & port ②, substituting there in 13, 14 and 15, we get,

$$b_1 = \frac{a_3}{\sqrt{2}}, b_2 = \frac{a_3}{\sqrt{2}} \text{ and } b_3 = 0.$$

Let P_3 (corresponding to a_3) be the power at port 3. Then this power divides equally between ports ① and ② and in phase ie $P_1 = P_2$.

$$\text{But } P_3 = P_1 + P_2 = 2P_1 = 2P_2.$$

The amount of power coming out of port ① or port ② due to input at port ③

$$= 10 \log_{10} \frac{P_1}{P_3} = 10 \log_{10} \frac{P_1}{2P_1} = 10 \log_{10} \left(\frac{1}{2} \right)$$

$$= -10 \log_{10} 2 = -10(0.3010) \approx -3 \text{ dB}$$

Hence the power coming out of port ① or port ② is 3 dB down with respect to input power at port ③, hence the H-plane Tee is called as 3dB splitter.

Case: 2 :- $a_1 = a_2 = a, a_3 = 0$.

$$b_1 = \frac{a}{2} - \frac{a}{2} + \frac{1}{\sqrt{2}} a_3 = \frac{a_3}{\sqrt{2}} = 0,$$

$$b_2 = -\frac{a}{2} + \frac{a}{2} + \frac{1}{\sqrt{2}} a_3 = \frac{a_3}{\sqrt{2}} = 0$$

$$b_3 = \frac{a_1}{\sqrt{2}} = \frac{a_2}{\sqrt{2}} = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}}$$

The O/P at port 3 is addition of powers at port 1 & port 2 and these are added in phase.

4.4.4. S-MATRIX FOR MAGIC TEE.

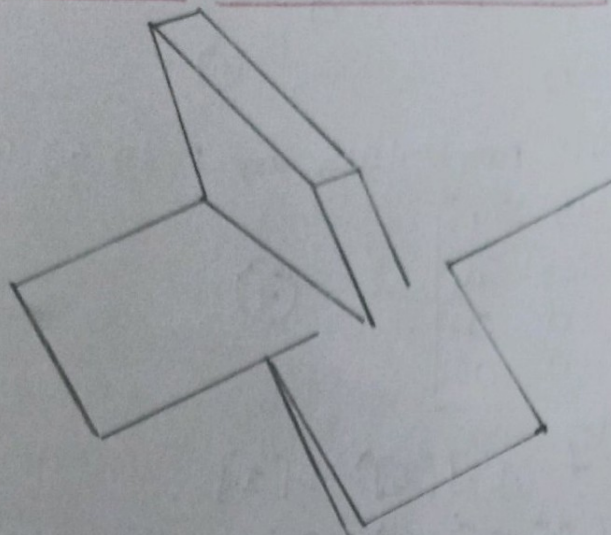


Fig:- MAGIC TEE.

Using the properties of E-H plane Tee, its scattering matrix can be obtained as follows:

1. $[S]$ is a 4×4 matrix since there are 4 ports.

$$\text{i.e. } [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \text{--- (1)}$$

2. Because of H-plane Tee section,

$$S_{23} = S_{13} \quad \text{--- (2)}$$

3. Because of E-plane Tee section,

$$S_{24} = -S_{14} \quad \text{--- (3)}$$

4. Because of geometry of the junction an input at port (3) cannot come out of port (4) since they are isolated ports and vice versa.

$$S_{34} = S_{43} = 0 \quad \text{--- (4)}$$

5. From Symmetric property, $S_{ij} = S_{ji}$.

$$S_{12} = S_{21}; S_{13} = S_{31}; S_{23} = S_{32}$$

$$S_{34} = S_{43}; S_{24} = S_{42}; S_{41} = S_{14} \quad \text{--- (5)}$$

6. If ports (3) and (4) are perfectly matched to the junction,

$$S_{23} = S_{44} = 0 \quad \text{--- (6)}$$

substituting the above properties eq 2 to 6, in eq (1), we get,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \quad \text{--- (7)}$$

7. From unitary property, $[S][S]^* \rightarrow [I]$.

$$\text{i.e. } \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 C_1 = |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad \text{--- 8}$$

$$R_2 C_2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad \text{--- 9}$$

$$R_3 C_3 = \quad \quad \quad |S_{13}|^2 + |S_{13}|^2 = 1 \quad \text{--- 10}$$

$$R_4 C_4 = \quad \quad \quad |S_{14}|^2 + |S_{14}|^2 = 1 \quad \text{--- 11}$$

From eq 10 and 11, $S_{13} = \frac{1}{\sqrt{2}} \quad \text{--- (12)} \quad S_{14} = \frac{1}{\sqrt{2}} \quad \text{--- (13)}$

Comparing eq (8) and (9), we get, $S_{11} = S_{22} \quad \text{--- (14)}$

Using these ~~equations~~ values from eq (12) and (13) in eq 8, we get,

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

$$S_{11} = S_{12} = 0 \quad \text{--- (14)}$$

i.e., from eq (7), $S_{22} = 0. \quad \text{--- (15)}$

This means that ports (1) and (2) are also perfectly matched to the junction. Hence in any four port junction if any two ports are perfectly matched to the junction, then the remaining two ports are automatically matched to the junction.