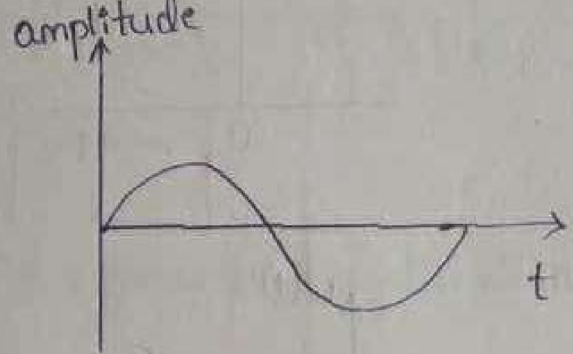
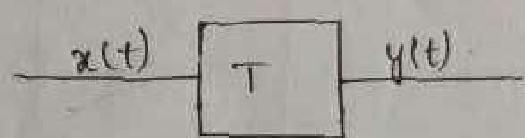
Signal : It is a graphical representation of some information.

Ex:



System: It is a physical device which produces a desired output with available input.



The block T is an operator which performs an operation on input to produce the output

$x(t)$ is called as excitation

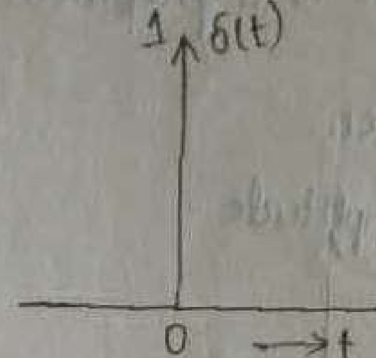
$y(t)$ is called as response.

$$\boxed{y(t) = T[x(t)]}$$

Some basic signals:

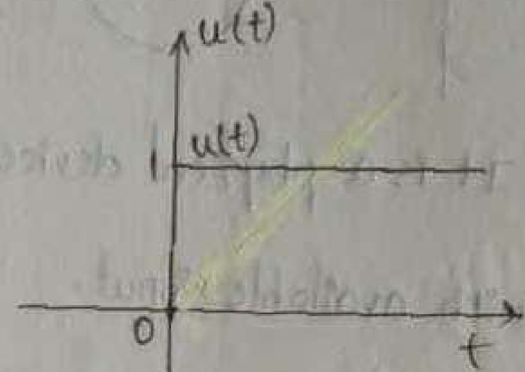
1. unit Impulse $\delta(t)$:-

$$\delta(t) = 1 \quad t = 0$$
$$= 0 \quad t \neq 0$$



2. unit step $u(t)$:-

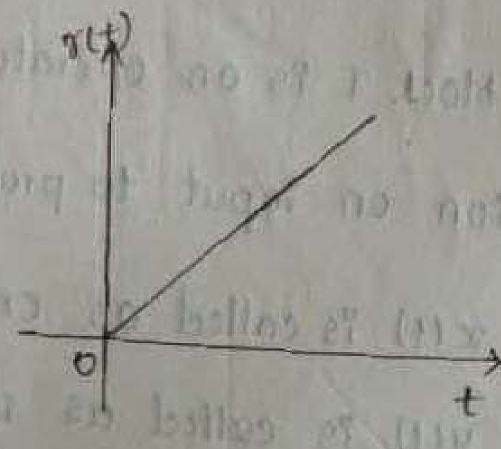
$$u(t) = 1 \quad t \geq 0$$
$$= 0 \quad t < 0$$



3. unit ramp $r(t)$:

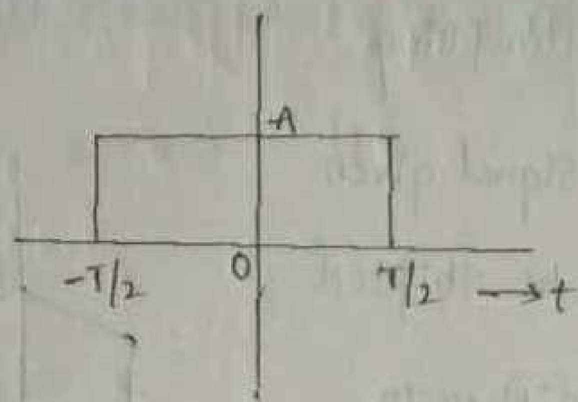
$$r(t) = At \quad t \geq 0$$
$$= 0 \quad t < 0$$

A = constant



Here unit represents the slope of the ramp

Rectangular (or) gate signal $\{A rect(t/T)\}$



A is Amplitude

Here T represents the duration

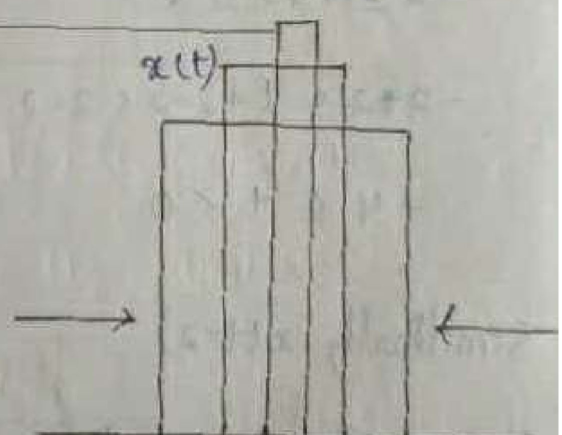
It is also represented as $A\Pi(t/T)$

Concept of Impulse (Dirac - Delta function)

height = ∞

Area = unity $\int \delta(t) = 1$

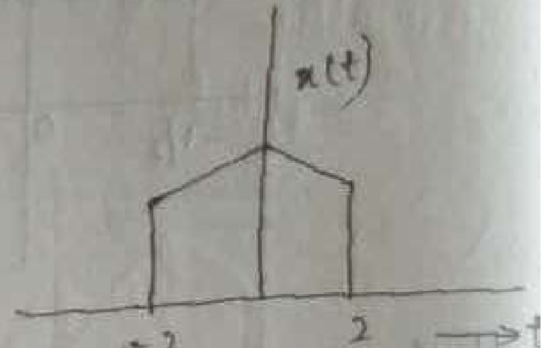
$\int x(t) \cdot dt = 1$



Working with signals :-

1. Shifting (or) Time shifting :-

Let $x(t)$ be the signal given. the signal $x(t)$ can be shifted on the time axis either to right or to left.



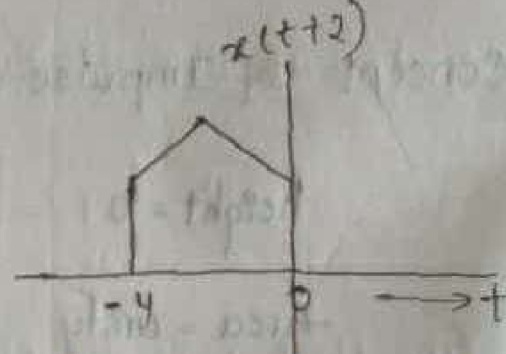
Consider $x(t+2)$

$-2 \leq t \leq 2$

$-2 \leq t+2 \leq 2$

$-2-2 \leq t+2-2 \leq 2-2$

$-4 \leq t \leq 0$

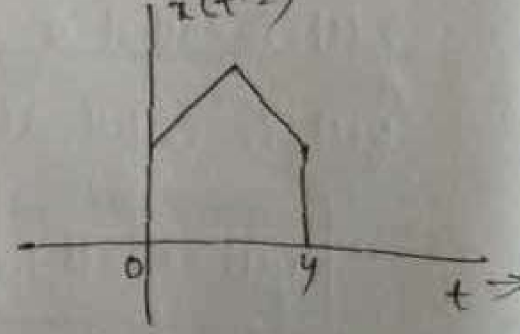


Similarly $x(t-2)$

$-2 \leq t \leq 2$

$-2 \leq t-2 \leq 2$

$0 \leq t \leq 4$



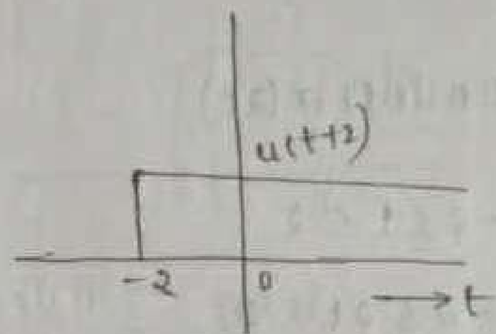
→ If $x(t)$ is a signal, then $x(t+t_0)$ shifted to left and $x(t-t_0)$ is shifted right.

→ consider $u(t+2)$

$0 \le t \le \infty$

$0 \le t+2 \le \infty$

$-2 \le t \le \infty$

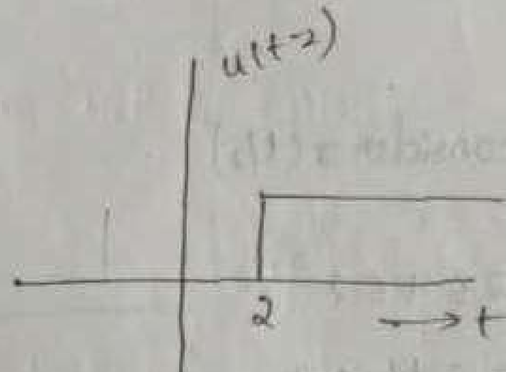


→ consider $u(t-2)$

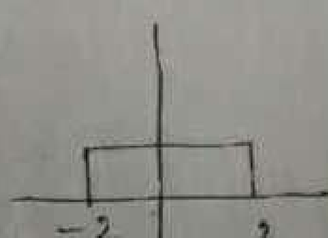
$0 \le t \le \infty$

$0 \le t-2 \le \infty$

$2 \le t \le \infty$



→ $u(t+2) - u(t-2)$



$= 1.\mathrm{Rec}(t/4)$

Time scaling :-

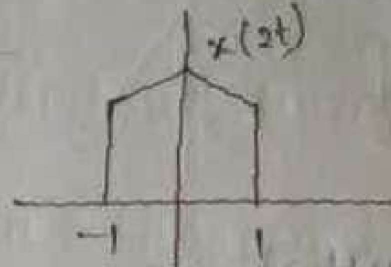
If $x(t)$ is a signal then $x(at)$ is its time scaled version. where a is constant

→ consider $x(2t)$

$-2 \le t \le 2$

$-2 \le 2t \le 2$

$-1 \le t \le 1$

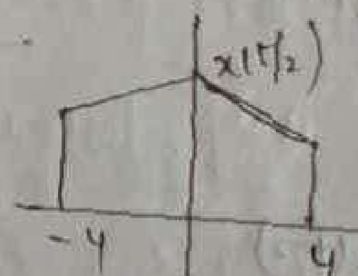


→ consider $x(t/2)$

$-2 \le t \le 2$

$-2 \le t/2 \le 2$

$-4 \le t \le 4$



The signal $x(at)$

→ Is compressed if $a > 1$

→ Is expanded if $a < 1$

Time reversal (or) folding

If $x(t)$ is a signal, $x(-t)$ is its folded version

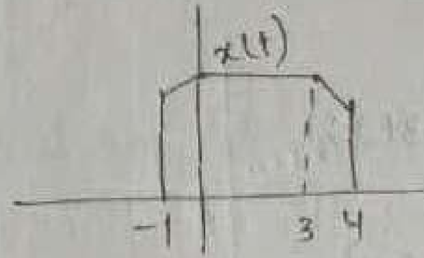
→ consider $x(t)$

$-1 \le t \le 4$

$-1 \le -t \le 4$

$1 \ge t \ge -4$

$-4 \le t \le 1$

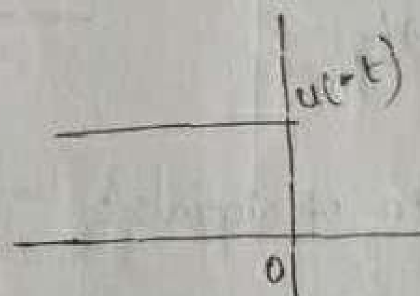


→ consider $u(-t)$

$0 \le t \le \infty$

$0 \le -t \le \infty$

$-\infty \le t \le 0$



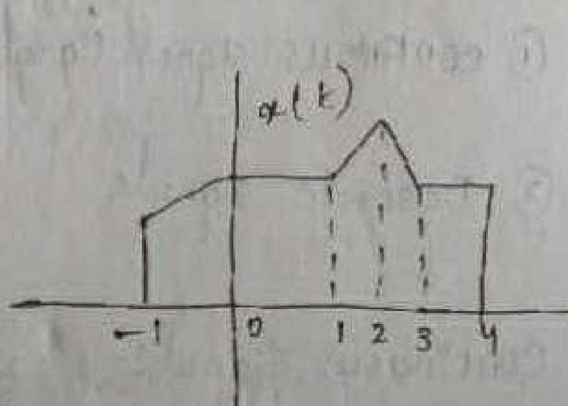
→ $x(-2t+4)$

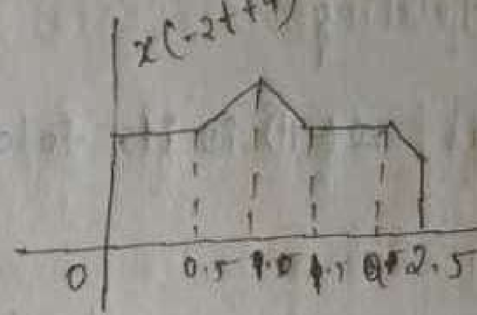
$-1 \le t \le 4$

$-1 \le -2t+4 \le 4$

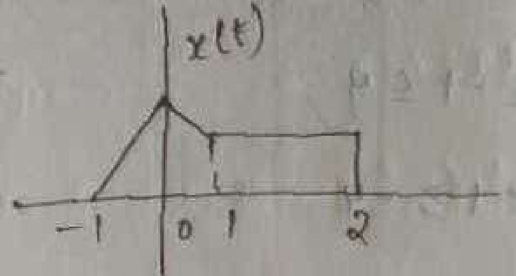
$-5 \le -2t \le 0$

$0 \le t \le 2.5$





$\rightarrow x(-3t+6)$



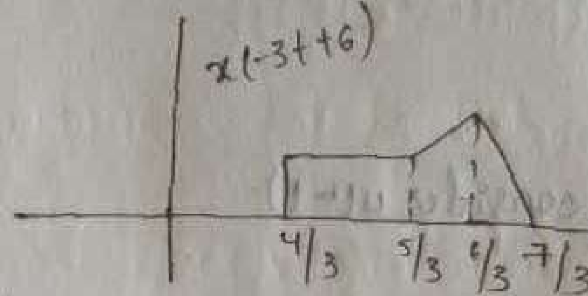
$-1 \leq t \leq 2$

$-1 \leq -3t+6 \leq 2$

$-7 \leq -3t \leq -4$

$7/3 \geq t \geq 4/3$

$4/3 \leq t \leq 7/3$



Classification of signals:

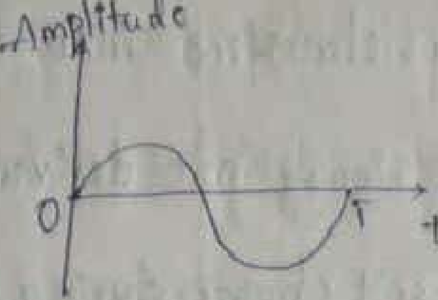
Signals are classified into two

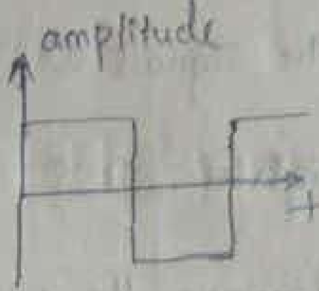
① continuous signals (analog)

② Descrete signals.

Continuous signal:-

A signal which has a value defined for every instant of time within specified interval

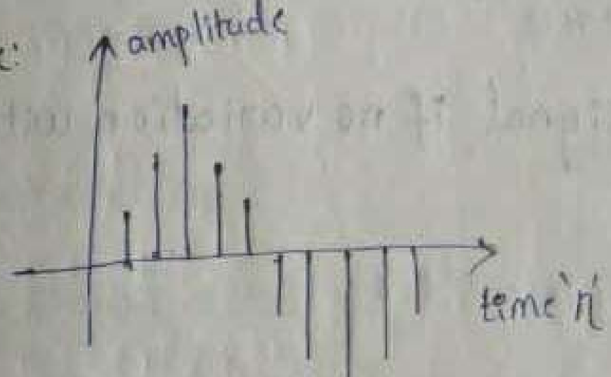
Ex:




Descrete signal:-

A signal which is defined only at specified or particular instant of time.

→ The distance between samples is decisided by by 'sampling Theorem'

Ex:


→ The descret time axis is represented by variable 'n', where 'n' is representing the index.

→ $x(n) = x(n\Delta T)$

where ΔT is the spacing between samples

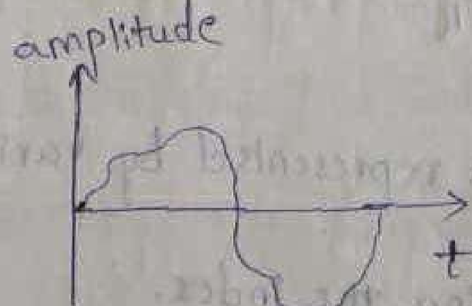
while n is the index which is always an integer

→ The signals are further classified into 5 catagories. since descrete signals are derived from continuous. The property doesn't change during the process.

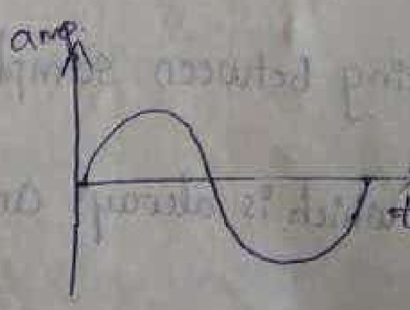
1. Deterministic and Random :-

Deterministic : A signal $x(t)$ is said to be deterministic if the variation with respect to 't' can be defined

Random signal :- A signal $x(t)$ is said to be not deterministic if random signal if no variation with respect to 't'



Ex. of deterministic :-



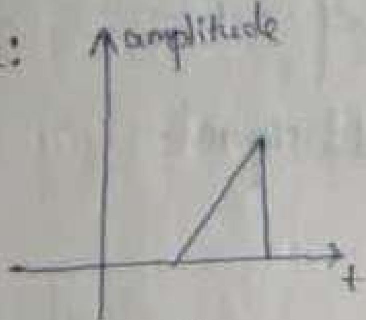
Causal (or) non-causal :-

A signal is said to be causal if it is defined only for positive values of times. where as a non-causal signal is defined for atleast some portion of negative time.

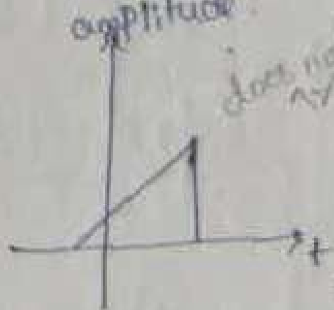
doesn't exist for -ve time ← causal signal is $x(t)=0$, for $t<0$

does not exist for +ve time ← Anti-causal signal is $x(t)=0$, for $t>0$

Ex:



causal



non-causal

The signal which exist for +ve as well as -ve time & neither causal nor anti-causal, it is non-causal signal

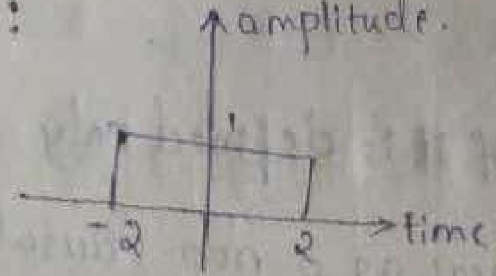
Even and odd :-

A signal $x(t)$ is said to be even if it satisfies the condition $x(t) = x(-t)$

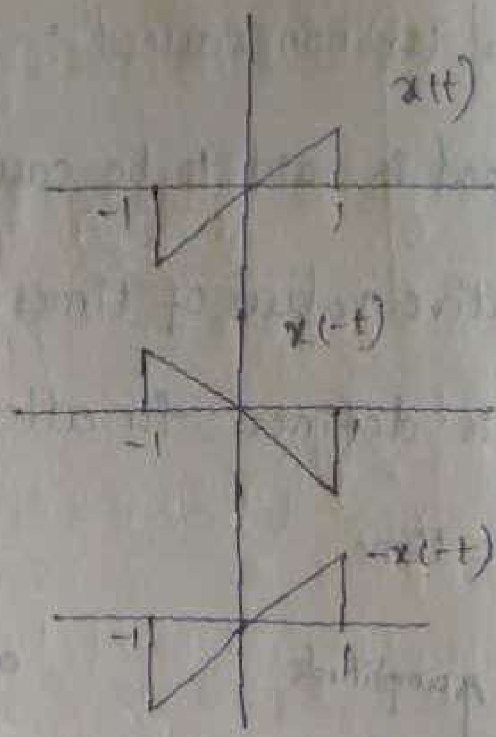
A signal $x(t)$ is said to be odd if it satisfies the condition $x(-t) = -x(t)$ (or) $x(t) = -x(-t)$

→ If a signal fails both the conditions then a signal is neither even nor odd.

Ex:



Even signal



odd signal.

Note :-

* All odd signals pass through origin and the convers need not be true.

Any signal $x(t)$ can be represented by the combination of even and odd.

$x(t)$

$x_o(t)$ $\quad$ $x_e(t)$

$$x(t) = x_o(t) + x_e(t) \quad \text{——} \quad (1)$$

$$x(-t) = x_e(-t) + x_o(-t)$$

$$x(-t) = x_e(t) - x_o(t) \quad \text{—— (2)}$$

$$(1)+(2) = x(t) + x(-t) = 2x_e(t)$$

$$x_e(t) = \frac{x(t)+x(-t)}{2}$$

$$(1)-(2) = x(t) - x(-t) = 2x_o(t)$$

$$x_o(t) = \frac{x(t)-x(-t)}{2}$$

$x(t) = e^{j2t}$

$x(-t) = e^{-j2t}$

$x_e(t) = \frac{e^{j2t}+e^{-j2t}}{2}$

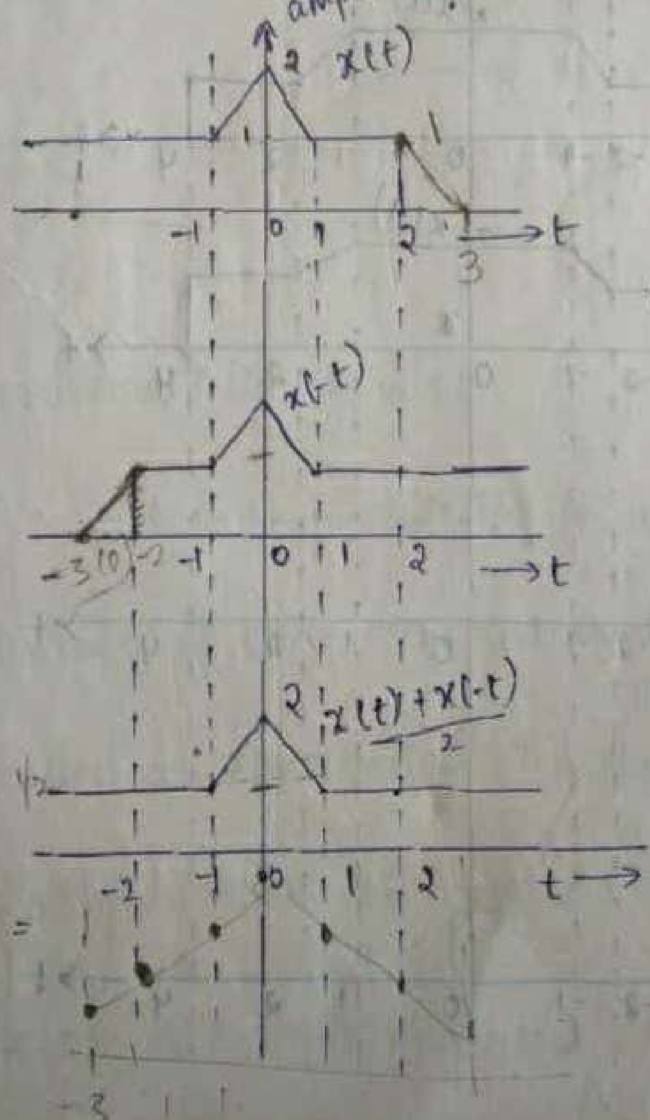
$= \cos 2t$

$x_o(t) = \frac{e^{j2t}-e^{-j2t}}{2}$

$j\sin 2t$

Example:

①



Even =

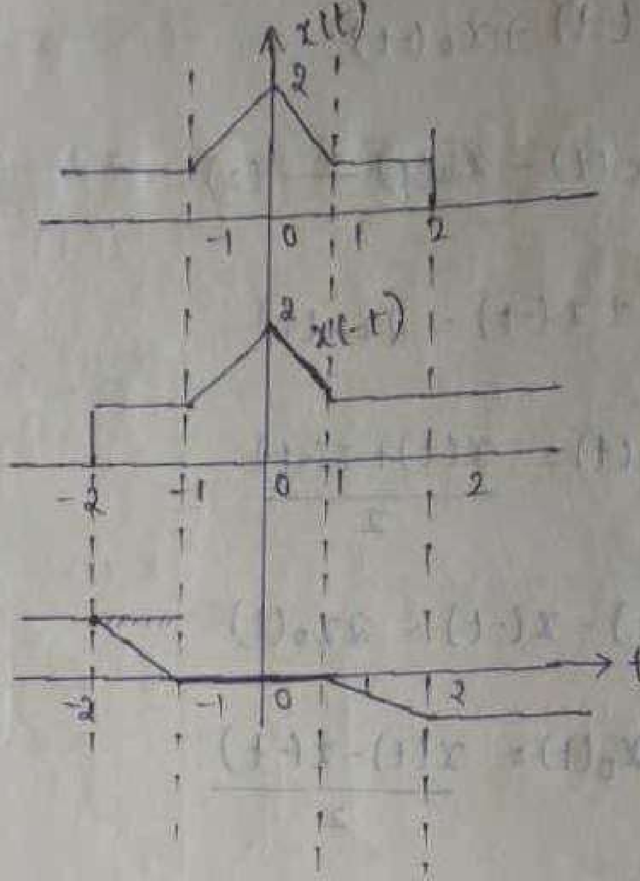
$\frac{x(t)+x(-t)}{2}$ =

$\frac{1+0}{2}$

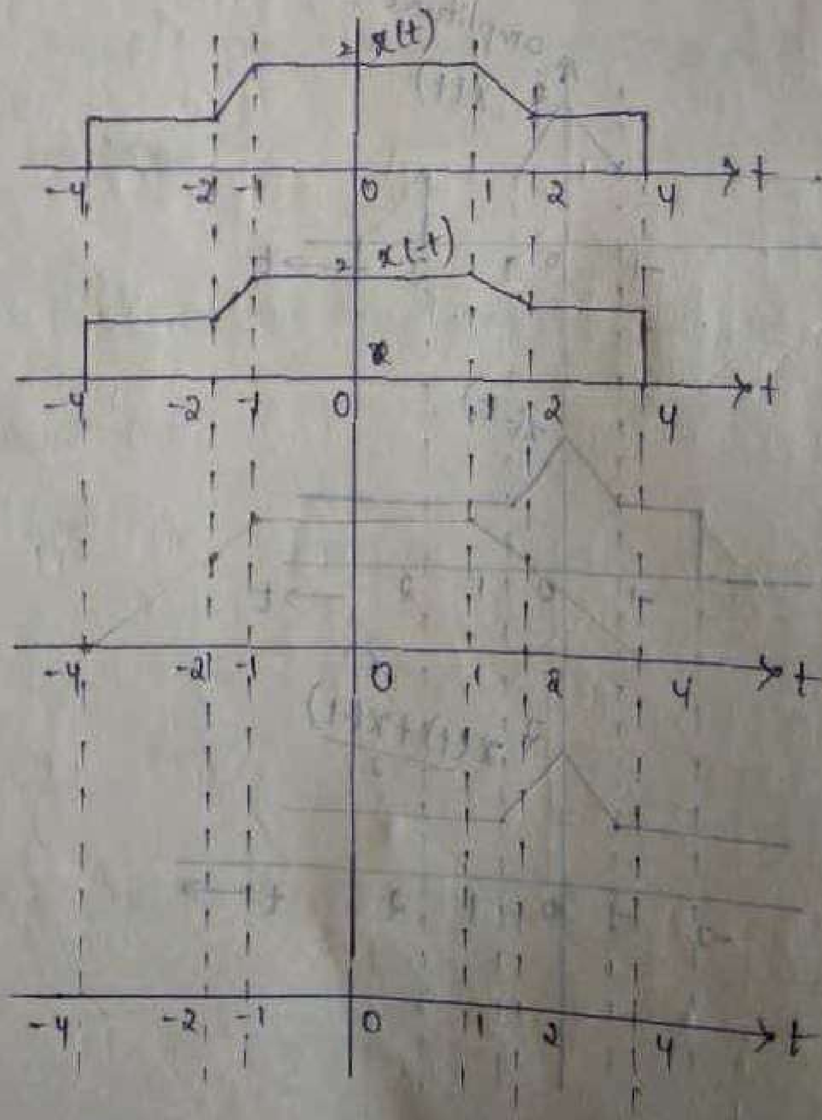
$\frac{1+1}{2}$

$\frac{1+0}{2}$

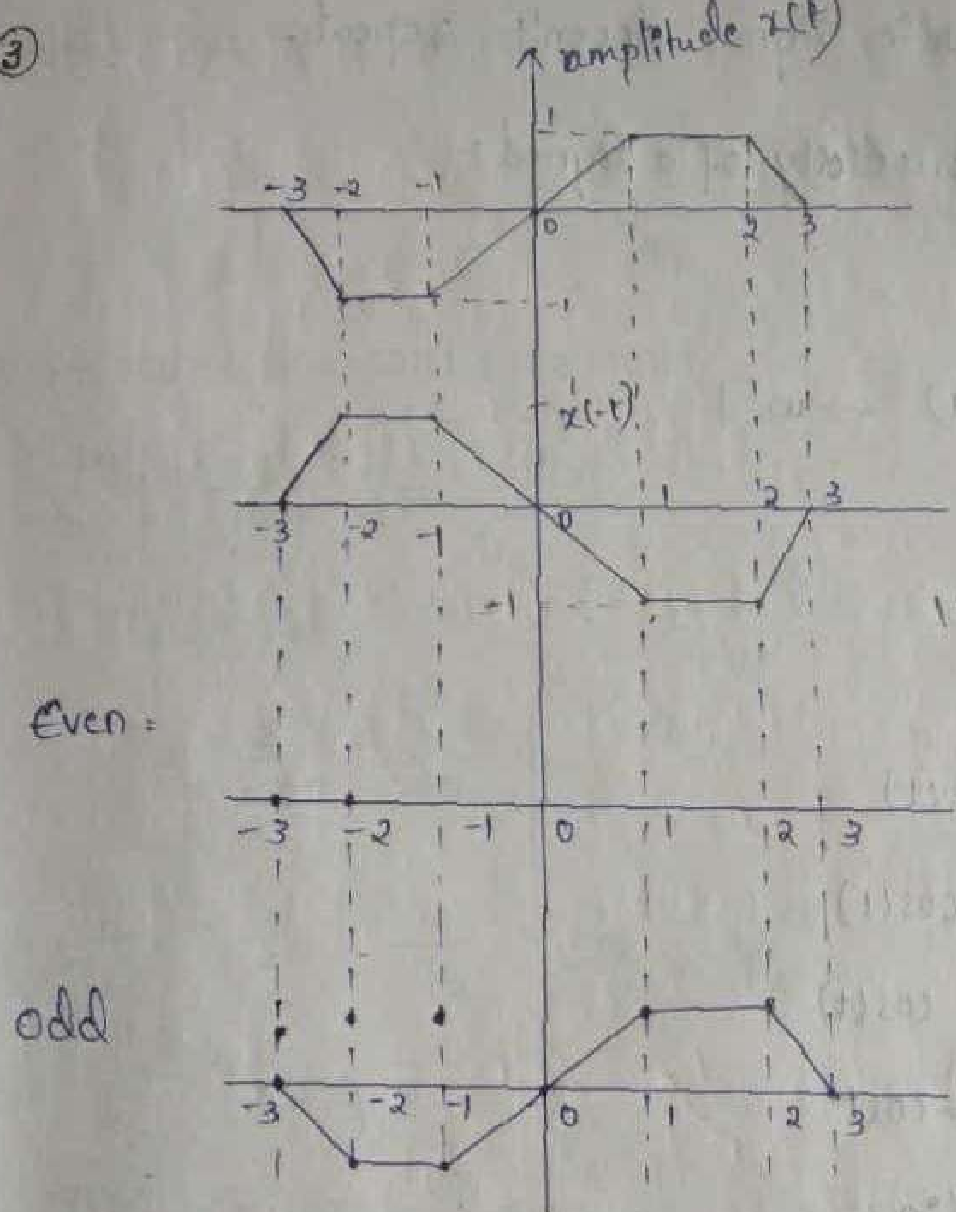
Odd:



(2)



③



Periodic and Aperiodic signals :-

→ A signal is said to be periodic if it repeats at regular intervals of time. The interval at which it repeats is called as Time period [T] Denoted by 'T'.

If $x(t)$ is a signal then this periodic if and only if

$x(t \pm kT) = x(t) \ \forall k$ k is integer(z)

→ An Aperiodic signal doesn't repeat.

· Test for periodicity of a signal :-

1. cos(t) :

$x(t) = \cos(t) \rightarrow \omega = 1$

$\omega = \frac{2\pi}{T}$

$$\boxed{T = \frac{2\pi}{\omega}}$$

$x(t+T) = x(t)$

$\cos(t+T) = \cos(t)$

$\cos(t+2\pi) = \cos(t)$

$\cos t = \cos t$

It is periodic.

2. $\cos 100\pi t + \sin 50\pi t$

$T_1 = \frac{2\pi}{100\pi} = \frac{1}{50}$

$T_2 = \frac{2\pi}{50\pi} = 1/25$

$\frac{T_1}{T_2} = \frac{1/50}{1/25} = \frac{1}{2}$ (it is rational so it is periodic)

$N = 2$

4. $\cos 100\pi t + \sin 50t$

$T_1 = \frac{2\pi}{100\pi} = 1/50$

$T_2 = \frac{2\pi}{50} = \pi/25$

$\frac{T_1}{T_2} = \frac{1}{2\pi}$ (presence of π treated as irrational & aperiodic)

Irrational, Aperiodic

5. $\cos^2(2\pi t)$

$\frac{1+\cos 2(2\pi t)}{2} = \cos^2(2\pi t)$

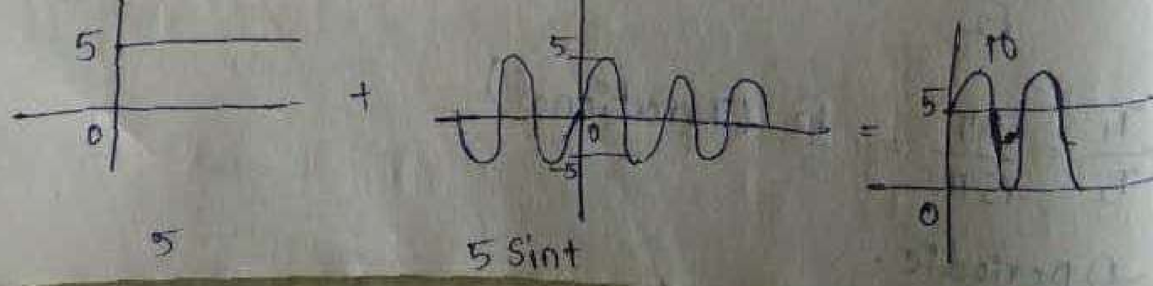
$\frac{1}{2} + \frac{\cos 4\pi t}{2}$

$T = \frac{2\pi}{4\pi} = 1/2$

rational, it is periodic.

Example:

$5 + 5\sin t$



total time period $(T) = T_1 \times N$

$$= \frac{1}{50} \times 2$$

$$T = 1/25$$

3. $\cos 100\pi t + \sin 50\pi t + \cos 150\pi t$

$$T_1 = \frac{2\pi}{100\pi} = 1/50$$

$$T_2 = 1/25$$

$$T_3 = \frac{2\pi}{150\pi} = \frac{1}{75}$$

$$\frac{T_1}{T_2} = \frac{1}{2} \qquad \frac{T_1}{T_3} = \frac{\frac{1}{50}}{1/75} = 3/2$$

$$N = 2$$

$$T = 6 \times 2 \times \frac{1}{50}$$

$$T = 1/25$$

4. $\cos t + \sin\sqrt{2}t$

$$T_1 = 2\pi/1 = 2\pi$$

$$T_2 = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$$

$$\frac{T_1}{T_2} = \frac{2\pi}{\sqrt{2}\pi} = \frac{\sqrt{2}}{2} \text{ (Irrational)}$$

Aperiodic.

→ DC has no impact on time period. Hence can be ignored.

6. $\cos t + \cos(t/3)$

$T_1 = 2\pi$

$T_2 = \frac{2\pi}{1/3} = 6\pi$

$\frac{T_1}{T_2} = \frac{2\pi}{6\pi} = 1/3 \quad N = 3$

$T = T_1 \times N$

$T = 6\pi$

Periodic

7. e^{-j2t}

$e^{-j2t} = \cos 2t - j\sin 2t$

$T_1 = \frac{2\pi}{2} = \pi$

$T_2 = \frac{2\pi}{2} = \pi$

$\frac{T_1}{T_2} = 1 \quad N = 1$

$T = 1 \times \pi$

$T = \pi$

Periodic

(8) $\sin \pi t \, u(t) \rightarrow$ Aperiodic

$\sin \pi t \Rightarrow \omega = \pi \Rightarrow T = \frac{2\pi}{\pi} = 2$

(periodic)

$u(t)$ exists only between $t = 0$ to $t = \infty$

Hence it is not periodic

(periodic signals exists for $-\infty < t < \infty$)

5. Energy and power signal:

→ A signal $x(t)$ is said to be an energy signal if its energy 'E' is finite. where E is given by

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \cdot dt \text{ joules} \quad 0 < E < \infty$$

non periodic signals are examples of energy signals

powersingnal: A signal $x(t)$ is said to be a power signal if power 'p' is finite, where p is given by

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \cdot dt \text{ watts} \quad 0 < P < \infty$$

periodic signals are examples of power signals

signals which fail both the above conditions are neither energy nor power signal.

Problems:-

1. $e^{-2t} u(t)$

$$x(t) = e^{-2t} u(t)$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \cdot dt$$

$$= \int_{-\infty}^{\infty} e^{-4t} \cdot u(t) \cdot dt$$

$u(t)$ limits 0 to ∞

$$= \int_0^\infty e^{-4t} \cdot dt$$

$$= \left[\frac{e^{-4t}}{-4}\right]_0^\infty$$

$$= \frac{-1}{4}\left[e^{-4(\infty)} - e^{-4(0)}\right]$$

$$= \frac{-1}{4}[0-1]$$

$$= 1/4 \text{ (finite)}$$

It is energy signal.

2 $e^{2t} u(-t)$

$x(t) = e^{-2t} u(-t)$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \cdot dt$$

$$E = \int_{-\infty}^{\infty} e^{4t} \cdot u(-t) \cdot dt$$

$$E = \int_{-\infty}^{0} e^{4t} \cdot dt$$

$$= \left(\frac{e^{4t}}{4}\right)_{-\infty}^{0} = 1/4 \text{ (finite) energy singnal}$$

3. $e^{-2t}u(t-5)$

$x(t) = e^{-2t}u(t-5)$

$0 \leq t-5 \leq \infty$

$5 \leq t \leq \infty$

$$E = \int_{-\infty}^{\infty} e^{-4t}\, u(t-5) \cdot dt$$

$$E = \int_{5}^{\infty} e^{-4t} \cdot dt$$

$$E = \frac{e^{-4(\infty)} - e^{-5(4)}}{-4}$$

$$E = \frac{e^{20}}{4}$$

4. $e^{-2t}u(t+4)$

$x(t) = e^{-2t}u(t+4)$

$$E = \int_{-\infty}^{\infty} \left|e^{-2t}u(t+4)\right|^2 \cdot dt$$

$0 \leq t \leq \infty$

$0 \leq t+4 \leq \infty$

$-4 \leq t \leq \infty$

$$= \int_{-\infty}^{\infty} e^{-4t} \cdot \underbrace{u(t+4)}_{-4 \text{ to } \infty} \cdot dt$$

$$= \int_{-4}^{\infty} e^{-4t} \cdot dt$$

$$= \left[\frac{e^{-4t}}{-4}\right]_{-4}^{\infty}$$

$$= \frac{e^{-\infty} - e^{16}}{-4}$$

$$= e^{16}/4$$

5. $e^{2t} u(-t+6)$

$$x(t) = e^{2t} u(-t+6)$$

$0 \le t \le \infty$

$0 \le -t+6 \le \infty$

$-6 \le -t \le \infty$

$6 \ge t \ge -\infty$

$-\infty \le t \le 6$

$$E = \int_{-\infty}^{\infty} |e^{2t} u(-t+6)|^2 \cdot dt$$

$$= \int_{-\infty}^{\infty} e^{4t} \cdot \underbrace{u(-t+6)}_{-\infty \text{ to } 6} \cdot dt$$

$$= \int_{-\infty}^{6} e^{4t} \cdot dt$$

$$= \left[\frac{e^{4t}}{4}\right]_{-\infty}^{6} = \frac{e^{4(6)} - 0}{4} = e^{24}/4$$

6. $e^{4t} \cdot u(-t-2)$

$0 \leq -t \leq \infty$

$x(t) = e^{4t} u(-t-2)$

$0 \leq -t-2 \leq \infty$

$2 \leq -t \leq \infty$

$-\infty \leq t \leq -2$

$$E = \int_{-\infty}^{\infty} e^{8t} \cdot \underbrace{u(-t-2)}_{-\infty \text{ to } -2} \cdot dt$$

$$= \int_{-\infty}^{-2} e^{8t} \cdot dt$$

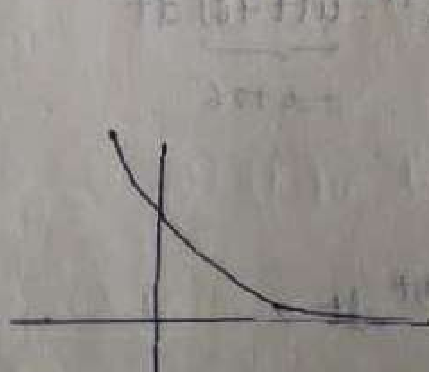
$$= \left[\frac{e^{8t}}{8}\right]_{-\infty}^{-2}$$

$$= \frac{e^{-16}}{8}$$

$$= \frac{1}{8 \cdot e^{16}}$$

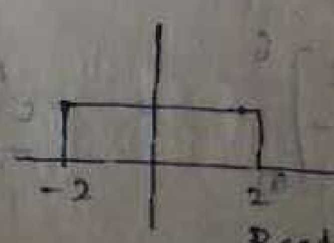
7. e^{-2t} Rect $(t/4)$

$$= \int_{-2}^{2} e^{-4t} \cdot dt$$



$$= \frac{-1}{4}\left[e^{-8} - e^{8}\right]$$

$$= \frac{1}{4}\left[2980 - \frac{1}{2980}\right]$$



Rcet $(t/4)$

$$= \frac{2980}{4}$$

$$= 745 J$$

8. $x(t) = u(t)$, check wheather energy signal (or) power signal.

$$e = \int_{-\infty}^{\infty} |u(t)|^2 . dt$$

$$= \int_{0}^{\infty} 1 . dt = \infty$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 . dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |u(t)|^2 . dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T/2} 1 . dt$$

$$= \lim_{T \to \infty} \frac{1}{\cancel{T}} \left(\cancel{T}/2\right]$$

$$P = \frac{1}{2} W$$

9. $x(t) = A\cos t$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \cdot dt$$

$$= \int_{-\infty}^{\infty} A^2 \cos^2 t \cdot dt$$

$$= A^2 \int_{-\infty}^{\infty} \frac{1+\cos 2t}{2} \cdot dt$$

$$= \frac{A^2}{2} \left[\int_{-\infty}^{\infty} 1 \cdot dt + \int_{-\infty}^{\infty} \cos 2t \cdot dt \right]$$

$$E = \infty$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 \cos^2 t \cdot dt$$

$$= \lim_{T \to \infty} \frac{A^2}{T} \int_{-T/2}^{T/2} \frac{1+\cos 2t}{2} \cdot dt$$

$$= \lim_{T \to \infty} \frac{A^2}{2T} \left[T + \int_{-T/2}^{T/2} \cos 2t \cdot dt \right]$$

$$= \lim_{T \to \infty} \frac{A^2}{2T} \times T + \lim_{T \to \infty} \frac{A^2}{2T} \int_{-T/2}^{T/2} \cos 2t \cdot dt$$

$P = \frac{A^2}{2}\omega$

$x(t) = A\cos t$ is a power signal.

Note:

1. $\underset{T\to\infty}{Lt} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 \cdot dt$

$P = \frac{1}{T}[E]$

→ power is time average of energy.

2. All power signals have energy $E = \infty$

3. All Energy signals have power $P = 0$

4. All periodic signals are power signals.

5. All finite duration signals are Energy signals.

Ex: $2\sin t \, rect(t/4)$.

$4\int_{-2}^{2} \sin^2 t \cdot dt \longrightarrow 4\int_{-2t}^{2} \frac{1-\cos 2t}{2} dt$

$= 2\left[(4) - \int_{-2}^{2} \cos 2t \cdot dt\right]$

$= 2\left[4 - \frac{1}{2}\{\sin 4 + \sin 4\}\right]$

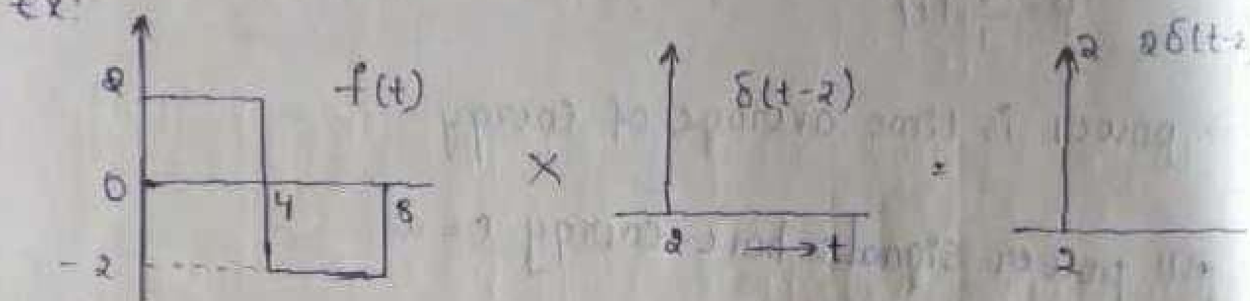
$= 2[4 - \underbrace{\sin 4}_{0.06}]$

$= 2[4]$

$= 8$

Integration involving impulse function.

ex:



$$\boxed{f(t) \times \delta(t-t_0) = f(t_0)\cdot\delta(t-t_0)}$$

→ The above property is called sampling or equivalence property where the impulse extracts a sample from the continuous signal.

Sifting property:-

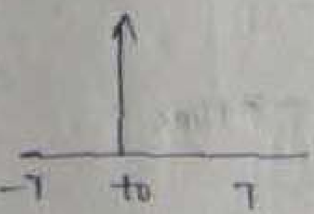
$$\int_{-T}^{T} f(t)\cdot\delta(t-t_0)\,dt = \underline{\qquad}$$

$$f(t) = \int_{-T}^{T} \delta(t-t_0)\cdot dt$$

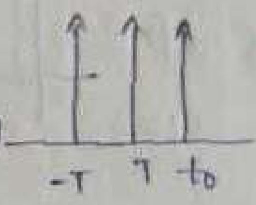
$$= f(t_0)\left[\int_{-T}^{T} \delta t \cdot dt - \int_{-T}^{T} \delta t_0 \, dt\right]$$

$$= f(t_0)\underbrace{\int_{-T}^{T} \delta(t - t_0) \cdot dt}_{1} = f(t_0)$$

$$= 0 \text{ else}$$



$= f(t_0)$



$= 0$

1. $\int_{-2}^{2} t^2 \cdot \delta(t-4) = 0$

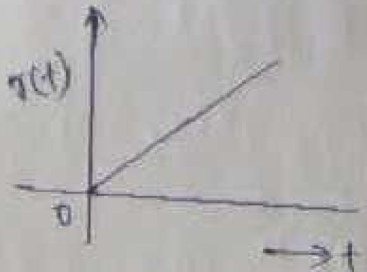
2. $\int_{-\infty}^{\infty} \cos t \; \delta(t-2) \cdot dt = \cos 2$

3. $\int_{-3}^{3} \cos t \; \delta(t-\pi) \cdot dt = 0$

4. $\int_{-3}^{4} \cos t \; \delta(t-\pi) \cdot dt = \cos \pi = -1$

Relation b/w signals:-

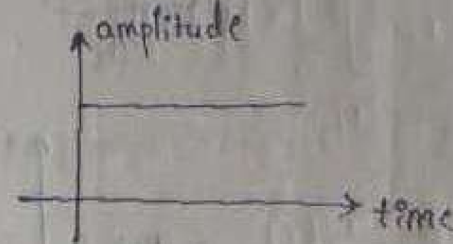
1.



$r(t) = t \ , \ t \geq 0$

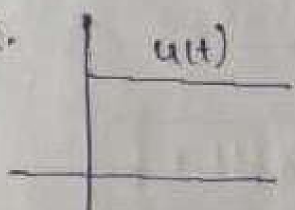
$\frac{d}{dt} r(t) = 1 \ , \ t \geq 0$

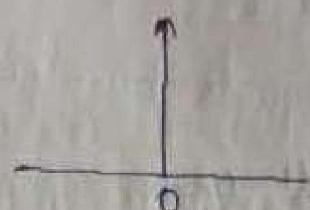
$= u(t)$



$\rightarrow \int u(t) \cdot dt = r(t)$

2.





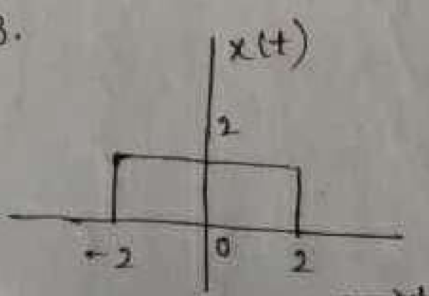
$\frac{d}{dt}(u(t))$

$\frac{d}{dt} u(t) = \delta(t)$

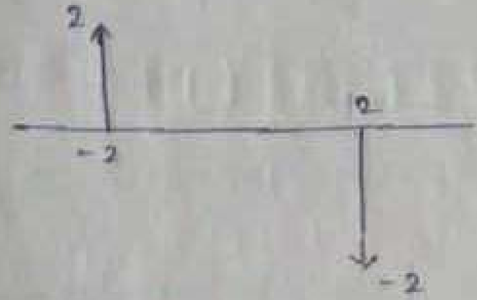
$\int \delta(t) = u(t)$

$$\boxed{\int \delta(t) = u(t)}$$

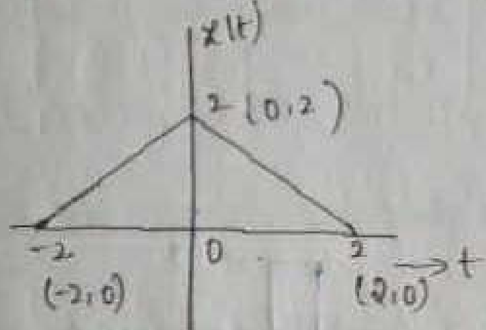
3.



$$\frac{d}{dt}x(t) = 2\delta(t+2) - 2\delta(t-2)$$



4.



$x(t) = t+2 \quad -2 \text{ to } 0$

$= -t+2 \quad 0 \text{ to } 2$

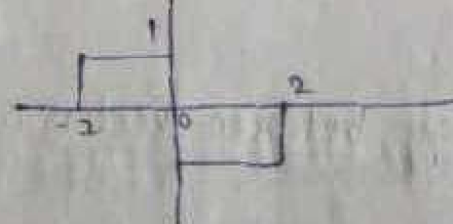
$y = mx$

$y = \cancel{2}x$

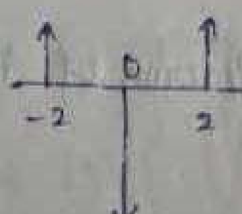
$y/x = 2$

$\frac{d}{dt}x(t) = 1 \quad -2 \text{ to } 0$

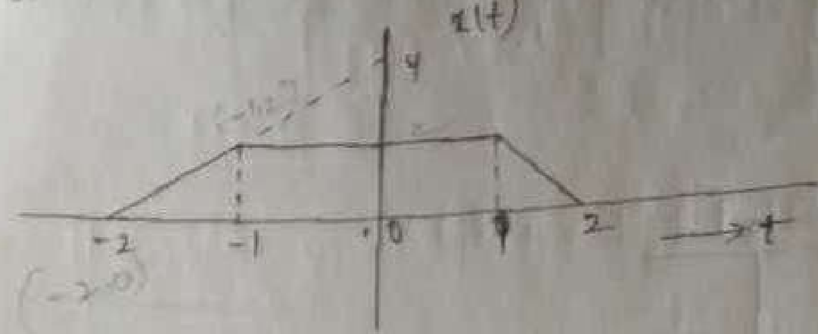
$= -1 \quad 0 \text{ to } 2$



$\frac{d^2}{dt^2}x(t) =$



$\delta(t+2) - 2\delta(t) + \delta(t-2)$



$x(t) = 4t + 8 \qquad -2 \text{ to } -1$

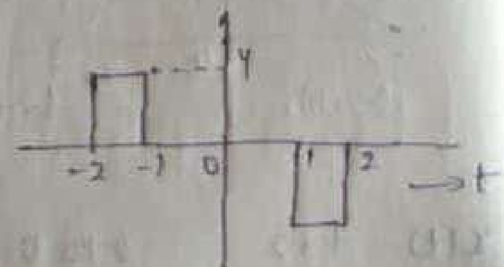
$\qquad 4 \qquad -1 \text{ to } 1$

$\qquad -4t+8 \qquad 1 \text{ to } 2$

$\frac{d}{dt}x(t) = 4 \qquad -2 \text{ to } -1$

$\qquad 0 \qquad -1 \text{ to } 1$

$\qquad -4 \qquad 1 \text{ to } 2$

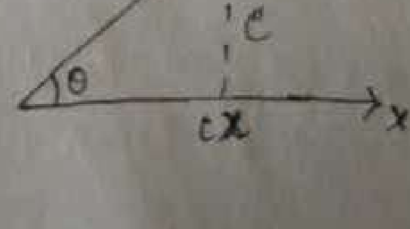


$$\frac{d^2}{dt^2}x(t) = 4\left[\delta(t+2) - \delta(t+1) - \delta(t-1) + \delta(t-2)\right]$$

Analogy between vectors and signals:

vector:

vectors are represented interms of it coordinates



consider the two vectors f and x, θ is the angle between f and x.

In the above figure cx is the component of vector 'f' along x. f can be expressed as $f = cx + e$.

e is minimum only when it is perpendicular to x.

The vector f and x are said to be orthogonal if the angle seperate them is 90° (or) If they are mutually orthogonal to each other. Any arbitary vector can be represented in terms of its unit vectors, provided they form a complete set. The vectors differ from each other only in terms of their co-efficents, while the representation remains same.

Orthogonality signals :-

just as vectors, we can also derive the orthogonality condition for signals. Assuming the two signals as f(t) and x(t), $f(t) = cx(t) + e(t)$

$$e(t) = f(t) - cx(t)$$

considering mean square error over the interval t_1 to t_2

$$\epsilon = \int_{t_1}^{t_2} (f(t) - cx(t))^2 \cdot dt$$

The value of c should be selected such that e is minimum, therefore we find $\frac{d\epsilon}{dc} = 0$

$$\frac{d\epsilon}{dc} \quad \epsilon = \frac{1}{(t_2-t_1)} \int_{t_1}^{t_2} f^2(t) \cdot dt + \int_{t_1}^{t_2} c^2 x^2(t)\, dt - \int_{t_1}^{t_2} 2 \cdot f(t) \cdot c x(t)$$

$$\frac{d\epsilon}{dc} = 0$$

$$2c \int_{t_1}^{t_2} x^2(t) \cdot dt - 2 \int_{t_1}^{t_2} f(t) \cdot x(t) \cdot dt = 0$$

$$c \int_{t_1}^{t_2} x^2(t) \cdot dt - \int_{t_1}^{t_2} f(t) \cdot x(t) \cdot dt = 0$$

$$C = \frac{\int_{t_1}^{t_2} f(t) \cdot x(t) \cdot dt}{\int_{t_1}^{t_2} x^2(t) \cdot dt} \longrightarrow (1)$$

for the signals to be orthogonal, c should be 0

~~in eq~~ substitude $c=0$ in $eq^n(1)$

$$\boxed{\int_{t_1}^{t_2} f(t)\cdot x(t)\cdot dt = 0}$$

eq:-

Test for orthogonality in the interval [0.1] for $f(t)=1$

$x(t)= \sqrt{3}\,(1-2t)$

$$\int_{t_1}^{t_2} f(t)\cdot x(t)\cdot dt = 0$$

$$\int_{0}^{1} 1\cdot \sqrt{3}\,(1-2t)\cdot dt = 0$$

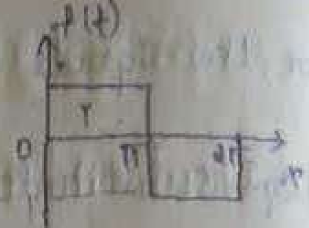
$$= \int_{0}^{1} \sqrt{3}\cdot dt - \sqrt{3}\cdot 2\int_{0}^{1} t\cdot dt$$

$$= \sqrt{3}\,[1-0] - \sqrt{3}\cdot 2\left[\frac{t^2}{2}\right]_0^1$$

$$= \sqrt{3} - \sqrt{3}$$

$$= 0$$

Q: Approximate the signal by sint.



$$f(t) = c\sin t$$

$$x(t) = \sin t$$

$$C = \frac{\int_0^{2\pi} f(t).x(t).dt}{\int_0^{2\pi} x^2(t).dt}$$

$$\int_0^{2\pi} f(t).x(t).dt = \int_0^{\pi} f(t).x(t).dt + \int_{\pi}^{2\pi} f(t).x(t).dt$$

$$= \int_0^{\pi} \sin t + (-1)\int_{\pi}^{2\pi} \sin t.dt$$

$$= \left[-\cos t\right]_0^{\pi} + \left[\cos t\right]_{\pi}^{2\pi}$$

$$= 4$$

$$\int_0^{2\pi} x^2(t).dt = \int_0^{2\pi} \frac{1-\cos 2t}{2}.dt$$

$$= \frac{1}{2}\int_0^{2\pi} (1-\cos 2t).dt$$

$$= \frac{1}{2}\left[2\pi - \left[\frac{\sin 2t}{2}\right]_0^{2\pi}\right]$$

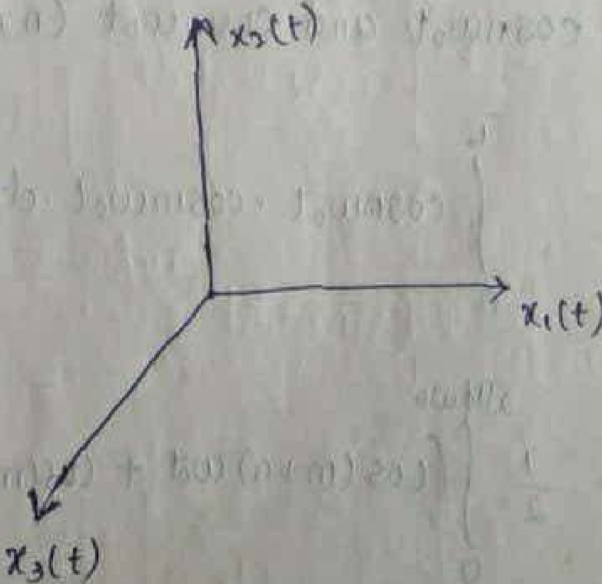
$$= \frac{1}{2}[2\pi - 0]$$

$$= \pi$$

$$\boxed{C = 4/\pi}$$

Orthogonal signal space:

just as vectors, given signals can be approximated using orthogonals. considering 3-dimentional space using 3 signals $x_1(t), x_2(t), x_3(t)$ [mutually orthogonal to each other] as shown.



Such signal space is called as orthogonal signal space.

closed (or) complete set of orthogonal functions :-

any orbitary function can be represented in terms of orthogonals provided their form a complete set

→ The set $\{x_1(t), x_2(t), \ldots x_n(t)\}$ is said to be a complete set of orthogonal function provided their exist no signal $x_m(t)$ for which the condition $\int x_n(t) \cdot x_m(t) = 0$ fails satisfies

check for the orthogonality of given signals over the interval $T_0 = 2\pi/\omega_0$

(i) $\cos n\omega_0 t$ and $\cos m\omega_0 t$ $(n \neq m)$

$$= \int_0^{T_0} \cos n\omega_0 t \cdot \cos m\omega_0 t \cdot dt$$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$$= \frac{1}{2}\int_0^{2\pi/\omega_0} \left(\cos(m+n)\omega_0 t + \cos(m-n)\omega_0 t\right) \cdot dt$$

$$= \frac{1}{2}\left[\frac{\sin(m+n)\cdot\omega_0 t}{m+n} + \frac{\sin(m-n)\omega_0 t}{m-n}\right]_0^{2\pi/\omega_0}$$

$= \frac{1}{2}[0+0]$

$= 0$

∴ It is orthogonal.

Q: $\sin n\omega_0 t$ and $\sin m\omega_0 t$ $(n \neq m)$, $T_0 = 2\pi/\omega_0$

$$= \int_0^{T_0} \sin n\omega_0 t \cdot \sin m\omega_0 t \cdot dt \qquad 2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$= \frac{1}{2}\int_0^{2\pi/\omega_0} [\cos(m-n)\cdot\omega_0 t - \cos(m+n)\cdot\omega_0 t]\cdot dt$$

$$= \frac{1}{2}\left[\frac{\sin(m-n)\cdot\omega_0 t}{m-n} - \frac{\sin(m+n)\cdot\omega_0 t}{m+n}\right]_0^{2\pi/\omega_0}$$

$= 0$

It is orthogonal.

$\sin n\omega_0 t$ and $\cos m\omega_0 t$

$$= \int_0^{T_0} \sin n\omega_0 t \cdot \cos m\omega_0 t \cdot dt \qquad 2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$= \frac{1}{2}\int_0^{2\pi/\omega_0} [\sin(m+n)\cdot\omega_0 t + \sin(m-n)\cdot\omega_0 t]\cdot dt$$

$$= -\frac{1}{2}\left[\left[\frac{\cos(n+m)\omega_0 t}{n+m}\right]_0^{2\pi/\omega_0} + \left[\frac{\cos(m-n)\omega_0 t}{m-n}\right]_0^{2\pi/\omega_0}\right]$$

$$= -\frac{1}{2}\left[1-1+1-1\right]$$

$$= 0$$

It is orthogonal.

4) k and $\cos n\omega_0 t$

$$= k\int_0^{T_0} \cos n\omega_0 t \cdot dt$$

$$= k\int_0^{2\pi/\omega_0} \cos n\omega_0 t \cdot dt$$

$$= k\left[\frac{\sin n\omega_0 t}{n}\right]_0^{2\pi/\omega_0}$$

$$= 0$$

Similarly k is also orthogonal to $\sin n\omega_0 t$.

Therefore ~~from~~ the above examples we can make a complete set of mutually orthogonal sinusoids as

$\{k, \cos\omega_0 t, \cos 2\omega_0 t, \ldots \cos n\omega_0 t, \sin\omega_0 t, \sin 2\omega_0 t \ldots \sin m\omega_0 t\}$

Trignometric fourier series :-

Fourier series is an approximation process where we represent any orbitary signal $x(t)$ in terms of orthogonals. T.F.S is a FS where the orthogonals selected are sinusoids.

Expression for TFS :-

$$\{k, \cos\omega_0 t, \ldots \cos n\omega_0 t, \sin\omega_0 t, \ldots \sin m\omega_0 t\} \quad \text{—(1)}$$

$$x(t) = k + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \qquad [k = a_0]$$

Evaluating the co-effients :-

$$\int_0^{T_0} x(t)\cdot dt = \left[\int_0^{T_0} a_0 \cdot dt + \sum_{n=1}^{\infty} a_n \int_0^{T_0} \cos n\omega_0 t \cdot dt + \sum_{n=1}^{\infty} b_n \int_0^{T_0} \sin n\omega_0 t \, dt\right]$$

$$\int_0^{T_0} x(t)\cdot dt = a_0 T_0 + 0 + 0$$

$$\boxed{a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\cdot dt}$$

a_k : finding a_n :-

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$\int_0^{T_0} x(t) \cdot \cos n\omega_0 t \cdot dt = \underbrace{\int_0^{T_0} a_0 \cdot \cos n\omega_0 t \cdot dt}_{0} + \int_0^{T_0} \sum_{n=1}^{\infty} a_n \int_0^{T_0} \cos^2 n\omega_0 t \cdot dt$$

$$+ \sum_{n=0}^{\infty} b_n \underbrace{\int_0^{T_0} \sin n\omega_0 t \cos n\omega_0 t \, dt}_{0}$$

$$\int_0^{T_0} x(t) \cdot \cos n\omega_0 t \cdot dt = a_n \int_0^{T_0} \frac{1 + \cos 2n\omega_0 t}{2} \cdot dt$$

$$= \frac{a_n}{2} \left[T_0 + \underbrace{\frac{\sin 2n\omega_0 t}{2n\omega_0}}_{0} \right]_0^{2\pi/\omega_0}$$

$$\int_0^{T_0} x(t) \cdot \cos n\omega_0 t \cdot dt = \frac{a_n T_0}{2}$$

$$\boxed{a_n = \frac{2}{T_0} \int_0^{T_0} x(t) \cdot \cos n\omega_0 t \cdot dt}$$

finding bn :-

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$\int_0^{T_0} x(t)\cdot \sin n\omega_0 t \cdot dt = \int_0^{T_0} a_0 \cdot \sin n\omega_0 t \cdot dt + \sum_{n=1}^{\infty} a_n \int_0^{T_0} \cos n\omega_0 t \cdot \sin n\omega_0 t \, dt$$

$$+ \sum_{n=1}^{\infty} b_n \int_0^{T_0} \sin^2 n\omega_0 t \cdot dt$$

$$\int_0^{T_0} x(t)\cdot \sin n\omega_0 t \cdot dt = b_n \int_0^{T_0} \frac{1-\cos 2n\omega_0 t}{2} \cdot dt$$

$$\int_0^{T_0} x(t)\cdot \sin n\omega_0 t \cdot dt = \frac{b_n}{2}\left[T_0 - \boxed{\frac{\sin 2n\omega_0 t}{2n\omega_0}}\right]_0^{2\pi/\omega_0}$$

$$\downarrow$$
$$0$$

$$= \frac{b_n T_0}{2}$$

$$\boxed{b_n = \frac{2}{T_0} \int_0^{T_0} x(t)\cdot \sin n\omega_0 t \cdot dt}$$

Deriving exponential fourier series from trignometric

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$= a_0 + \sum_{n=1}^{\infty} a_n \left[\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} \right] + \sum_{n=1}^{\infty} b_n \left[\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j} \right]$$

$$= a_0 + \sum_{n=1}^{\infty} \frac{e^{jn\omega_0 t}}{2}(a_n - jb_n) + \frac{e^{-jn\omega_0 t}}{2}(a_n + jb_n)$$

$$c_n = \frac{a_n - jb_n}{2} \qquad c_{-n} = \frac{a_n + jb_n}{2}$$

$$a_0 + \sum_{n=1}^{\infty} e^{jn\omega_0 t} c_n + e^{-jn\omega_0 t} c_{-n}$$

$$a_0 = c_0$$

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}}$$

finding c_n!

$$c_n = \frac{a_n - jb_n}{2}$$

$$= \frac{1}{2}\left[\frac{2}{T}\int_{\langle T \rangle} f(t)\cos n\omega_0 t\, dt - j\frac{2}{T}\int_{\langle T \rangle} f(t)\cdot \sin n\omega_0 t\, dt \right]$$

$$= \frac{1}{T}\int_{\langle T \rangle} f(t)\cdot(\cos n\omega_0 t - j\sin n\omega_0 t)\cdot dt$$

$$\boxed{C_n = \frac{1}{T}\int_{\langle T\rangle} f(t)\cdot e^{-jn\omega_0 t}\cdot dt}$$

Component trignometric Rounes series Representation.

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \cos n\omega_0 t$$

$$a_n = r_n \cos\theta_n$$

$$b_n = r_n \sin\theta_n$$

$$\text{Where } r_n = \sqrt{a_n^2 + b_n^2}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} r_n \cos\theta_n \cos n\omega_0 t + r_n \sin\theta_n \cos n\omega_0 t$$

$$\boxed{f(t) = a_0 + \sum_{n=1}^{\infty} r_n\left(\cos(n\omega_0 t + \theta_n)\right)}$$

$$\boxed{\theta_n = \tan^{-1}\left(b_n/a_n\right)}$$

Relation between TFS and EFS coefficients

$$C_n = \frac{1}{2}(a_n - jb_n)$$

$$|C_n| = \frac{1}{2}\sqrt{a_n^2 + b_n^2}$$

$$\boxed{|C_n| = \frac{1}{2} r_n}$$

Dirichlet's condition :

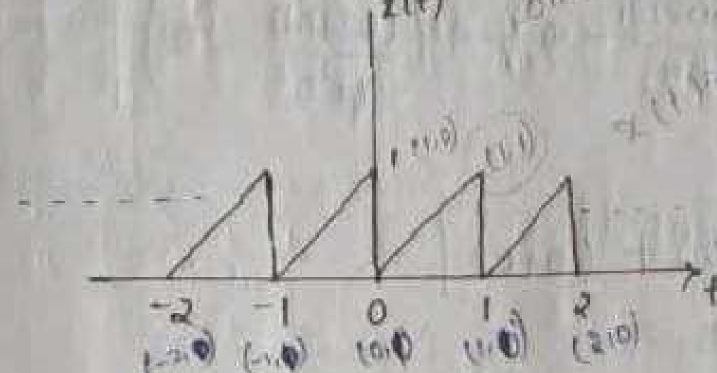
These are the conditions for the existance of fourier series.

1. The signal must be periodic
2. The signal must be absolutely integrable
3. The signal should have finite no of discontinuity
4. The signal should have finite no of maximum and minimum values

Condition 1 and 2 are called 'strong Dirichlet's' conditions. while 3 and 4 are weak dirichlet's' conditions

→ All the physical signal satisfy conditions 3 & 4 find the fourier Series coefficient for given signal

(i)



Here $x(t) = t$, $T_0 = 1$, $\omega_0 = 2\pi / T_0 = 2\pi$

$$a_0 = \frac{1}{T_0} \int_0^{T_0} x(t) \cdot dt$$

$$= \frac{1}{1} \int_0^1 t \, dt$$

$$= 1/2$$

$$a_n = \frac{2}{T_0} \int_0^{T_0} x(t) \cdot \cos n\omega_0 t \cdot dt$$

$$= \frac{2}{1} \int_0^1 t \cdot \cos 2n\pi t \cdot dt$$

$$= 2 \left\{ t \cdot \int_0^1 \cos 2n\pi t \cdot dt - \int_0^1 \frac{\sin 2n\pi t}{2n\pi} \right]$$

$$= 2\left[\left(t \cdot \frac{\sin 2n\pi t}{2n\pi}\right)_0^1 + \left(\frac{\cos 2n\pi t}{4n^2\pi^2}\right)_0^1\right]$$

$$= 2\left[\left(\frac{1 \cdot \sin 2n\pi}{2n\pi} - 0\right) + \left(\frac{\cos 2n\pi}{4n^2\pi^2} - \cancel{0}^{\,1}\right)\right]$$

$$= 2\left[(0-0) + (\overset{0}{0}+0)\right]$$

$\sin 2n\pi = 0$

$$a_n = 0$$

$$b_n = \frac{2}{T_0}\int_0^{T_0} x(t) \cdot \sin n\omega_0 t \cdot dt$$

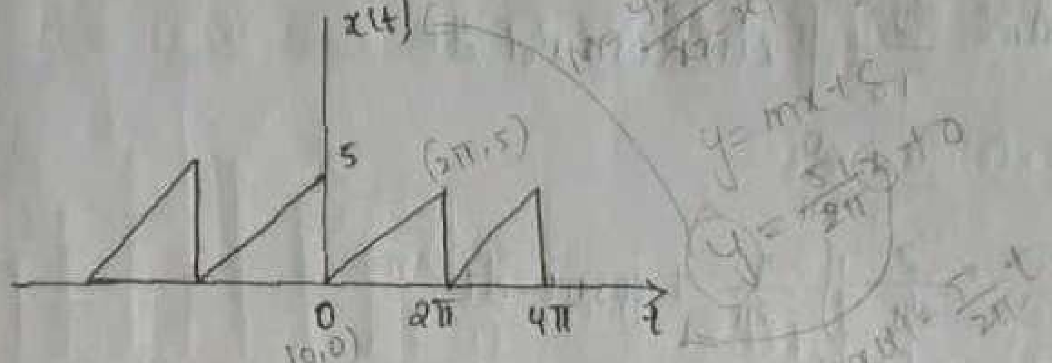
$$= \frac{2}{1}\int_0^1 t \cdot \sin n2\pi t\, dt$$

$$= 2\left[t \cdot \int \sin n2\pi t\, dt + \int_0^1 \frac{\cos n2\pi t\, dt}{n}\right]$$

$$= 2\left[\left(-\frac{t\cos n2\pi t}{2n\pi}\right)_0^1 + \left(\frac{\sin n2\pi t}{4n^2\pi^2}\right)_0^1\right]$$

$$= 2\left[\left(\frac{-1\cos 2n\pi}{2n\pi} + 0\right) + \frac{1}{4n^2\pi^2}[0-0]\right]$$

$$b_n = -1/n$$



$x(t) = \text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1} \cdot t$ $\qquad \omega_0 = \dfrac{2\pi}{T_0}$

$x(t) = \dfrac{5}{2\pi} t$ $\qquad = 2\pi/2\pi = 1$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t) \cdot dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{5}{2\pi} \cdot t \, dt$$

$$= \frac{5}{4\pi^2}\int_0^{2\pi} t \, dt$$

$$= \frac{5}{4\pi^2}\left[\frac{t^2}{2}\right]_0^{2\pi}$$

$$= \frac{5}{4\pi^2}\left[\frac{4\pi^2}{2}\right]$$

$$a_0 = 5/2$$

$$a_n = \frac{2}{T_0}\int_0^{T_0} x(t) \cdot \cos n\omega_0 t \cdot dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi} \frac{5}{2\pi} \cdot t \cos nt \cdot dt$$

$$= \frac{5}{2\pi^2}\int_0^{2\pi} t \cos nt \cdot dt$$

$$= \frac{5}{2\pi^2}\left[t \cdot \int \cos nt \cdot dt - \int_0^{2\pi} 1 \cdot \int \cos nt \cdot dt \right]$$

$$= \frac{5}{2\pi^2}\left[\left[\frac{t \cdot \sin nt}{n}\right]_0^{2\pi} - \int_0^{2\pi} \frac{\sin nt}{n} \cdot dt \right]$$

$$= \frac{5}{2\pi^2}\left[\left(\frac{2\pi \sin 2n\pi - 0}{n}\right) + \left(\frac{\cos nt}{n^2}\right)_0^{2\pi} \right]$$

$$= \frac{5}{2\pi^2}\left[0 + \frac{\cos 2n\pi - \cos 0}{n^2} \right]$$

$$= \frac{5}{2\pi^2}\left[0 + 1 - 1 \right]$$

$$a_n = 0$$

$$b_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cdot \sin n\omega_0 t \cdot dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi} \frac{5}{2\pi}\cdot t \cdot \sin nt \cdot dt$$

$$= \frac{5}{2\pi^2}\left[t\cdot \int \sin nt\cdot dt + \int_0^{2\pi} \frac{\cos nt}{n}\cdot dt \right]$$

$$= \frac{5}{2\pi^2}\left[\left(t\left[-\frac{\cos nt}{n} \right] \right)_0^{2\pi} + \left(\frac{\sin nt}{n^2} \right)_0^{2\pi} \right]$$

$$= \frac{5}{2\pi^2}\left[\left(\frac{-2\pi \cos 2n\pi - 0}{n} \right) + \left(\frac{\sin 2n\pi - \sin 0}{n^2} \right) \right]$$

$$= \frac{5}{2\pi^2}\left[\frac{-2\pi}{n} + 0 \right]$$

$$b_n = \frac{-5}{n\pi}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$x(t) = 2.5 + \sum_{n=1}^{\infty} \frac{-5}{n\pi} \sin nt$$

Exponential series

$C_0 = a_0 = 2.5$

$$C_n = \frac{1}{T}\int_0^T f(t)\cdot e^{-jn\omega_0 t}\cdot dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{5}{2\pi} t\cdot e^{-jnt}\cdot dt$$

$$= \frac{5}{4\pi^2}\left[\int_0^{2\pi} t\cdot e^{-jnt}\cdot dt \right] \qquad \left[\because \int t\cdot e^{at}\cdot dt = \frac{e^{at}}{a^2}(at-1)\right]$$

$$= \frac{5}{4\pi^2}\left[\frac{e^{-jnt}}{(-jn)^2}\left[-jnt-1\right] \right]_0^{2\pi} \qquad \left[j^2 = -1\right]$$

$$= \frac{5}{-4\pi^2 n^2}\left\{ e^{-j2n\pi}\left[-j2n\pi - 1\right] - 1(-1) \right\}$$

$$= \frac{-5}{4\pi^2 n^2}\left\{ 1(-j2n\pi - 1 + 1) \right\}$$

$$= \frac{5}{2n\pi}(j)$$

Relation b/w exponential and trignometric Series

$$C_n = \frac{a_n - j b_n}{2}$$

$$\left|\frac{a_n}{2} - \frac{j b_n}{2}\right| = \left| j \frac{5}{2n\pi} \right|$$

$$a_n = 0, \quad b_n = \frac{-5}{2n\pi}$$

Spectrum:

It is a graph with frequency on x-axis. Since the fourier series coefficents are complex, they have magnitude and phase, therfore they have magnitude spectrum and phase spectrum.

for the above problem

$$r_n = \sqrt{a_n^2 + b_n^2}$$

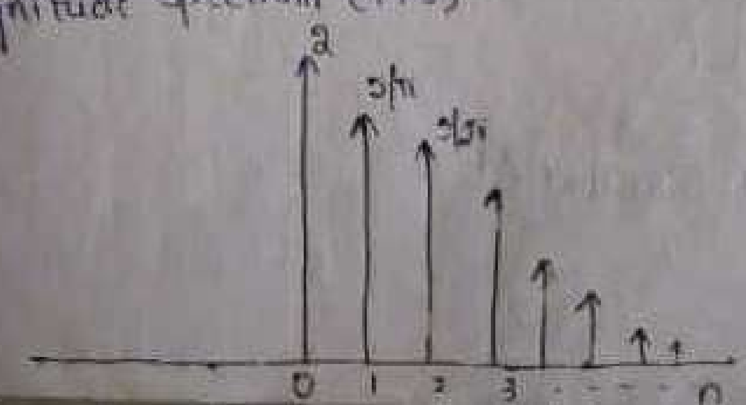
w_0 = fundamental frequency

n = index of ... coefficient of w_0

$a_n = 0$

$\therefore r_n = b_n$

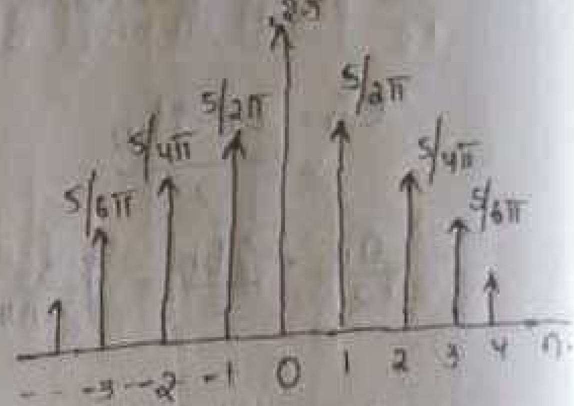
Magnitude spectrum (TFS)



magnitude
frequency Spectrum (EFS)

$C_n = \frac{j5}{2n\pi}$

$|C_n| = \frac{5}{2n\pi}$



Type of signal	a_0	a_n	b_n
even	Exist	exist	0
odd	0	0	Exist

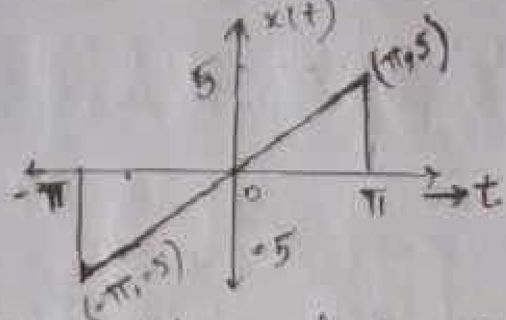
$a_0 = \frac{1}{T}\int_0^T f(t)\cdot dt$ if Even → Exist
odd → 0

$a_n = \frac{2}{T}\int_0^T \overset{\text{Even}}{f(t)\cdot \cos n\omega_0 t}\cdot dt$ if even × even = exist

$b_n = \frac{2}{T}\int_0^T \overset{\text{odd}}{f(t)\cdot \sin n\omega_0 t}\cdot dt$ if even × odd = 0

odd × odd = even

Q:



Since it is odd signal bn exist

$$x(t) = \frac{5}{\pi} t \qquad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

$$b_n = \frac{2}{2\pi} \int_{-\pi}^{\pi} \frac{5}{\pi} t \cdot \sin n \cdot t \, dt$$

$$= \frac{10}{\pi^2} \int_{0}^{\pi} t \cdot \sin nt \cdot dt$$

$$= \frac{10}{\pi^2} \left[t \cdot \int \sin nt \, dt + \int \frac{d}{dt}(t) \cdot \frac{\cos nt}{n} \right]_0^{\pi}$$

$$= \frac{10}{\pi^2} \left[-\frac{t \cos nt}{n} + \frac{\sin nt}{n^2} \right]_0^{\pi}$$

$$= \frac{10}{\pi^2} \left[\left(\frac{-\pi \cos n\pi}{n} + 0 \right) - (0 - 0) \right]$$

$$= \frac{-10 \, (-1)^n}{n\pi} = \frac{10 \, (-1)^{n+1}}{n\pi}$$

$a_0 = 0$

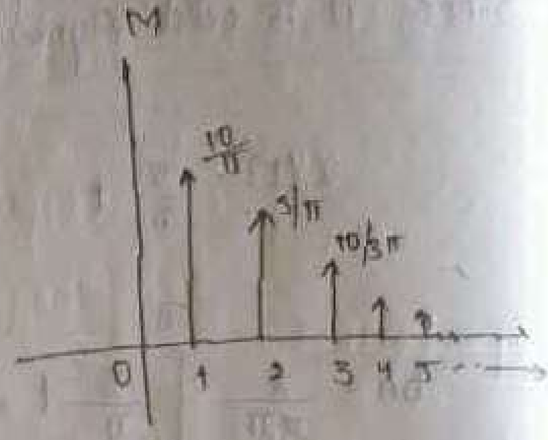
$a_n = 0$

$b_n = \frac{-10(-1)^n}{n\pi}$

$r_n = \sqrt{a_n^2 + b_n^2}$

$r_n = b_n$

$r_n = \frac{10}{n\pi}$

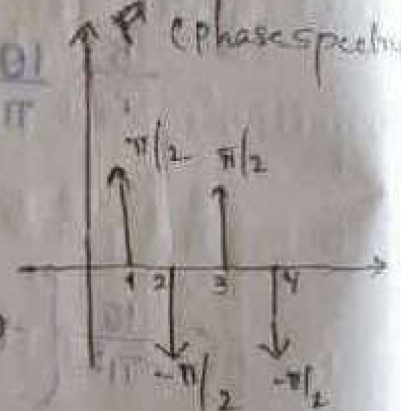


$b_n = \frac{10(-1)^{n+1}}{n\pi}$

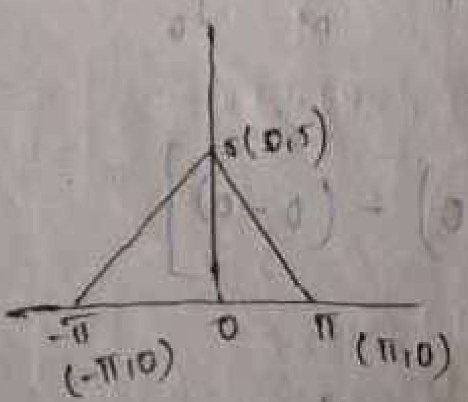
$\tan^{-1}(b_n/a_n)$ b_n is +ve, n-odd $+\infty$, $+\pi/2$

b_n is -ve, n-even

$-\infty$, $-\pi/2$



Q.



$$f(t) = \frac{5}{\pi}t + 5 \quad , -\pi \text{ to } 0$$

$T = 2\pi$

$\omega_0 = 1$

$$= \frac{-5}{\pi}t + 5 \quad , 0 \text{ to } \pi$$

Since it is a even signal a_0, a_n is exist, $b_n = 0$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\cdot dt$$

$$= \frac{2}{2\pi}\int_0^{\pi} f(t)\cdot dt$$

$$= \frac{1}{\pi}\int_0^{\pi}\left(\frac{-5}{\pi}t + 5\right)\cdot dt$$

$$= \frac{1}{\pi}\left[-\frac{5}{\pi}\frac{t^2}{2} + 5t\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{5\cancel{\pi}^2}{2\cancel{\pi}} + 5\pi\right]$$

$$= \frac{1}{\pi}\left[-\frac{5\pi}{2} + 5\pi\right] = \frac{5\pi}{2\pi} = 5/2$$

$$a_n = \frac{2}{T_0}\int_{-\pi}^{\pi} f(t)\cdot \cos n\omega_0 t \cdot dt$$

$$= \frac{4}{2\pi}\int_{0}^{\pi}\left(\frac{-5}{\pi}t + 5\right)\cdot \cos n\omega t \cdot dt$$

$$= \frac{2}{\pi}\int_{0}^{\pi}\left(\frac{-5}{\pi}t\cdot \cos nt + 5\cos nt\right)\cdot dt$$

$$= \frac{2}{\pi}\left[\int_{0}^{\pi}\frac{-5}{\pi}t\cdot \cos nt\, dt + \int_{0}^{\pi} 5\cos nt\cdot dt\right]$$

$$= \frac{2}{\pi}\left[\frac{-5}{\pi}\int_{0}^{\pi} t\cdot \cos nt\cdot dt + 5\int_{0}^{\pi}\cos nt\cdot dt\right]$$

$$= \frac{2}{\pi}\left[\frac{-5}{\pi}\left\{t\cdot\frac{\sin nt}{n} + \frac{\cos nt}{n^2}\right\}_{0}^{\pi} + 5\left\{\frac{\sin nt}{n}\right\}_{0}^{\pi}\right]$$

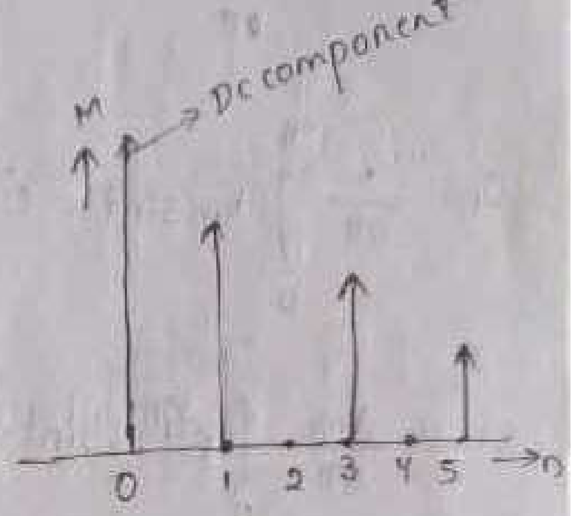
$$= \frac{2}{\pi}\left[\frac{-5}{\pi}\left\{\pi\frac{\sin n\pi}{n} + \frac{\cos n\pi}{n^2} - \frac{(0+1)}{n^2}\right\} + \frac{5}{n}\left\{\sin n\pi - \sin 0\right\}\right]$$

$$= \frac{2}{\pi}\left[\frac{-5}{\pi}\left\{0\times\pi + \frac{(-1)^n}{n^2} - \frac{(0+1)}{n^2}\right\} + \frac{5}{n}\left\{0 - 0\right\}\right]$$

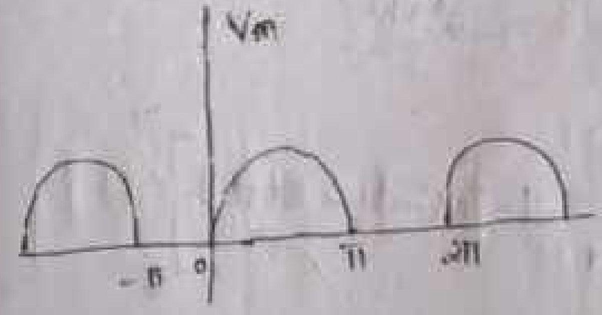
$$= \frac{-10(-1)^n}{\pi^2\cdot n^2} \qquad \frac{-10}{\pi^2}\left(\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right)$$

NOTE

For even signals, since $b_n = 0$, the phase spectrum is zero.



Q



find the exponential series.

$f(t) = V_m \sin t \cdot dt$

$$C_0 = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin t \cdot dt$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m \sin t \cdot dt$$

$$= \frac{-V_m}{2\pi}\left[\cos t\right]_0^{\pi}$$

$$C_0 = \frac{-V_m}{2\pi}[-2]$$

$$C_0 = \frac{V_m}{2\pi}$$

$$C_n = \frac{1}{2\pi}\int_0^{\pi} V_m \sin t \cdot e^{-jnt} \cdot dt$$

$$= \frac{V_m}{2\pi}\int_0^{\pi} e^{-jnt} \cdot \sin t \cdot dt$$

$$\therefore \int e^{ax} \cdot \sin bx \cdot dx = \frac{e^{ax}}{a^2+b^2}\left[a \sin bx - b\cos bx\right]$$

$$= \frac{V_m}{2\pi}\left[\frac{e^{-jnt}}{(-jn)^2+1}(-jn\sin t - \cos t)\right]_0^{\pi}$$

$$= \frac{V_m}{2\pi}\left[\frac{e^{-jnt}}{1-n^2}(-jn\sin t - \cos t)\right]_0^{\pi}$$

$$= \frac{V_m}{2\pi}\left[\frac{e^{-jn\pi}}{1-n^2}(-jn\sin\pi - \cos\pi) - \frac{e^{0}}{1-n^2}(-jn\sin 0 - \cos 0)\right.$$

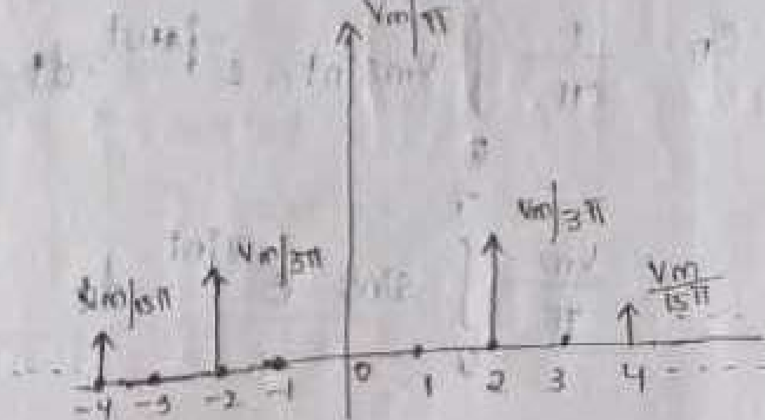
$$= \frac{V_m}{2\pi}\left[\frac{e^{-jn\pi}}{1-n^2}(0+1) - \frac{1}{1-n^2}(0-1)\right]$$

$$= \frac{V_m}{(1-n^2)2\pi}\left[\cos n\pi - j\sin n\pi + 1\right]$$

$$e^{-jn\pi} = \cos n\pi - j\sin n\pi$$

$$= \frac{V_m}{(1-n^2)2\pi}\left[(-1)^n + 1\right]$$

Magnitude spectrum :-



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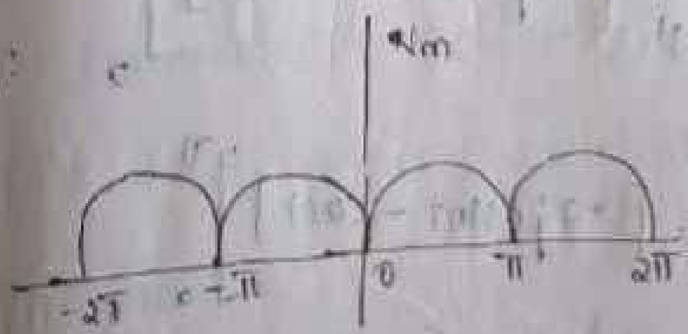


$$f(t) = V_m \sin t \cdot dt \qquad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$$

$$C_0 = \frac{1}{\pi}\int_0^{\pi} V_m \sin t \cdot dt$$

$$= \frac{V_m}{\pi}\int_0^{\pi} \sin t \cdot dt$$

$$\frac{V_m}{\pi}\left[-\cos t\right]_0^{\pi}$$

$$= \frac{-V_m}{\pi}\left[-1-1\right]$$

$$C_0 = \frac{2V_m}{\pi}$$

$$C_n = \frac{1}{\pi}\int_0^{\pi} V_m \sin t \cdot e^{-j2n\omega t} \cdot dt$$

$$= \frac{V_m}{\pi}\int_0^{\pi} \sin t \cdot e^{-2jnt} \cdot dt$$

$$= \frac{V_m}{\pi}\left[\frac{e^{-2jnt}}{(-2nj)^2+(1)^2}\left[-2jn\sin t - 1.\cos t\right]\right]_0^{\pi}$$

$$= \frac{V_m}{\pi}\left[\frac{e^{-2jnt}}{1-4n^2}\left[-2jn\sin t - \cos t\right]\right]_0^{\pi}$$

$$= \frac{V_m}{\pi(1-4n^2)}\left[e^{-2jn\pi}\left[-2jn\sin\pi - \cos\pi\right] - e^{0}\left[-2jn\sin 0 - \cos 0\right]\right]$$

$$= \frac{V_m}{\pi(1-4n^2)}\left[\cos 2n\pi - j\sin 2n\pi\left[0-1\right] - 1\left[0-1\right]\right]$$

$$= \frac{V_m}{(1-4n^2)\pi}\left[1 - 0(-1) + 1\right]$$

$$= \frac{2V_m}{(1-4n^2)\pi}$$

Spectrum:-

NOTE:

For Exponential fourier series, all magnitude spectra possess Even symmetry while phase spectra possess odd symmetry

Properties of fourier series :-

1. Linearity property :-

If $x_1(t)$ has fourier series of $X_1(k)$

$$x_1(t) \xleftrightarrow{FS} X_1(k)$$

$$x_2(t) \xleftrightarrow{FS} X_2(k)$$

then $a x_1(t) + b x_2(t) \longleftrightarrow a X_1(k) + b X_2(k)$

Proof :

$$X(k) = \frac{1}{T} \int_{\langle T \rangle} x(t) \cdot e^{-jn\omega_0 t} \cdot dt$$

$$= \frac{1}{T}\int_{\langle T\rangle}\left(a x_1(t) + b x_2(t)\right)\cdot e^{-jn\omega_0 t}\cdot dt$$

$$= \frac{1}{T}\,a\cdot\int_{\langle T\rangle} x_1(t)\cdot e^{-jn\omega_0 t}\cdot dt + \frac{1}{T}\,b\cdot\int_{\langle T\rangle} x_2(t)\cdot e^{-jn\omega_0 t}$$

$$X(k) = aX_1(k) + bX_2(k)$$

2. Time shifting property :-

$$x(t) \xleftrightarrow{FS} X(k)$$

$$x(t-t_0) \xleftrightarrow{FS} e^{-j\omega t_0} X(k)$$

$$\tau = t - t_0$$

$$t = \tau + t_0$$

$$= \frac{1}{T}\int_{\langle T\rangle} x(t-t_0)\cdot e^{-j\omega t}\,dt$$

$$= \frac{1}{T}\int_{\langle T\rangle} x(\tau)\cdot e^{-j\omega(\tau+t_0)}\cdot d\tau\,.$$

$$= \frac{1}{T}\int_{\langle T\rangle} x(\tau)\cdot e^{-j\omega\tau}\, e^{-j\omega t_0}\, d\tau$$

$$= e^{-j\omega t_0}\cdot X(k)$$

Representing $X(k)$ in terms of Magnitude (M) and phase (ϕ) $X(k) = M e^{j\phi}$, then

$$x(t-t_0) \xleftrightarrow{FS} e^{-j\omega t_0} \cdot X(k)$$

$$e^{-j\omega t_0} \cdot M \cdot e^{j\phi}$$

$$M \cdot e^{j(\phi - \omega t_0)}$$

It means shift in time domain effects the phase

Magnitude remains unchanged

3. Time scaling property :-

$$x(t) \xleftrightarrow{FS} X(k)$$

$$x(at) \xleftrightarrow{FS} X(k)$$

$$= \frac{1}{T} \int_{\langle T \rangle} x(at) \cdot e^{-jk\omega_0 t} \cdot dt$$

$$at = \tau$$

$$= \frac{1}{T} \int_{\langle T \rangle} x(\tau) \cdot e^{-jk\left(\frac{\omega_0}{a}\right)\tau} \cdot \frac{d\tau}{a}$$

$$= \frac{1}{T/a} \times a \int_{\langle T/a \rangle} x(\tau) \cdot e^{-jk(\omega_0/a)\cdot\tau} \cdot d\tau$$

$$= \frac{1}{T} \int_{\langle T/a \rangle} x(\tau) \cdot e^{-jk(\omega_0/a)\cdot\tau} \cdot d\tau$$

$= X(k)$, with the fundamental frequency as (ω_0/a)

4. Frequency shifting property :-

$$x(t) \xleftrightarrow{FS} X(k)$$

$$e^{jk_0\omega_0 t} x(t) \longleftrightarrow X(k-k_0)$$

$$= \frac{1}{T} \int_{\langle T \rangle} x(t) \cdot e^{jk_0\omega_0 t} \cdot e^{-jk\omega_0 t} \cdot dt$$

$$= \frac{1}{T} \int_{\langle T \rangle} x(t) \cdot e^{-j(k-k_0)\omega_0 t} \cdot dt$$

$$= X(k-k_0)$$

Parseval's theorem for power

According to this theorem, the power in time domine is equal to power in frequency domine.

That means if a signal $x(t)$ as a power of 'p' watts then the sum of powers collected from all the frequency components of $x(k)$ is also equal to p. In simple it is an example of law of conservation of energy.

Therefore $x(t) \longleftrightarrow x(k)$

$$P = \frac{1}{T}\int_{\langle T\rangle} |x(t)|^2 \cdot dt = \sum_{k=-\infty}^{\infty} |x(k)|^2$$

The power of the signal is given by

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 \cdot dt.$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t) \cdot x^*(t) \cdot dt$$

Since $x(t)$ and $x^*(t)$ are same with respect to magnitude.

$$C_n = X(k)$$

$$x(t) = \sum_{k=-\infty}^{\infty} X(k) \cdot e^{jk\omega_0 t}$$

$$x^*(t) = \sum_{k=-\infty}^{\infty} x^*(k) \cdot e^{-jk\omega_0 t}$$

$$= \cancel{\lim_{T\to\infty}} \cancel{\frac{1}{T}} \int_{-T/2}^{T/2} \cancel{x(k) \cdot e^{jk\omega_0 t}} \cdot \cancel{x^*(k) \cdot e^{-jk\omega_0 t}} \cdot dt$$

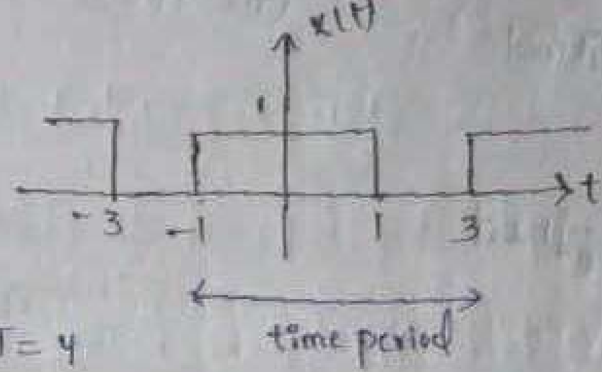
$$= \lim_{T\to\infty} \frac{1}{T} \int_{\langle T \rangle} x(t) \cdot \sum_{k=-\infty}^{\infty} x^*(k) \cdot e^{-jk\omega_0 t} \cdot dt$$

$$= \lim_{T\to\infty} \sum_{k=-\infty}^{\infty} x^*(k) \frac{1}{T} \underbrace{\int_{\langle T \rangle} x(t) \cdot e^{-jk\omega_0 t} \cdot dt}_{X(k)}$$

$$= \sum_{k=-\infty}^{\infty} x^*(k) \cdot X(k)$$

$$= \sum_{k=-\infty}^{\infty} |X(k)|^2$$

Find the power up to second harmonic for the given signal



$T = 4$

$\omega_0 = 2\pi/4 = \pi/2$

$$P = \frac{1}{4}\int_{-1}^{1} 1 \cdot dt$$

$$P = 1/2$$

power in frequency domain

$$C_n = X(k) = \frac{1}{4}\int_{-1}^{1} 1 \cdot e^{-jk\frac{\pi}{2}t} \cdot dt$$

$$= \frac{1}{4}\left[\frac{e^{-jk\pi/2\, t}}{-jk\pi/2}\right]_{-1}^{1}$$

$$= \frac{1}{4}\left[\frac{e^{-j\pi/2 k} - e^{jk\pi/2}}{-jk\pi/2}\right]$$

$$= \frac{1}{4}\left[\frac{e^{jk\pi/2} - e^{-jk\pi/2}}{jk\pi/2}\right]$$

$$= \frac{1}{4jk\pi/2}\left[\cos k\pi/2 + j\sin k\pi/2 - (\cos k\pi/2 - j\sin k\pi/2)\right]$$

$$= \frac{\cancel{j}\sin k\pi/2}{\cancel{2j}\,k\pi/2}$$

$$= \frac{\sin k\pi/2}{k\pi}$$

$$x(k) = \frac{1}{k\pi}\left[\sin k\pi/2\right]$$

0 for k is even

$1/k\pi$ for k is odd

$$c_0 = x(0) = \frac{1}{T}\int_{\langle T\rangle} x(t)\,dt$$

$$= \frac{1}{4}\int_{-1}^{1} 1.dt$$

$$= 1/2$$

$$= \sum_{k=-\infty}^{\infty} |x(k)|^2$$

$$= \sum_{k=-2}^{2} |x(k)|^2$$

$$c_n = c_0,\ c_1,\ c_2,\ c_{-1},\ c_{-2}$$

$c_0 \to 1/4$, $c_1 \to 1/\pi^2$, $c_2 = 0$, $c_{-1} \to 1/\pi^2$, $c_{-2} = 0$

$$= 0.25 + \frac{1}{\pi^2} + \frac{1}{\pi^2}$$ $$\frac{0.45}{0.5} = 90\%$$

$$= 0.45$$

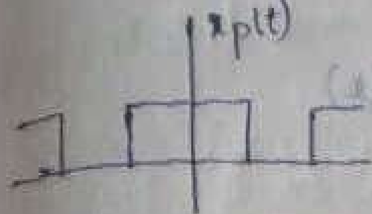
The total power upto second harmonic is 0.45 which is 90% of the total power. It means the remaining 10% of power is distributed among the infinite number of frequency components left. Therefore we consider the power up to the first few harmonics and leaving the rest.

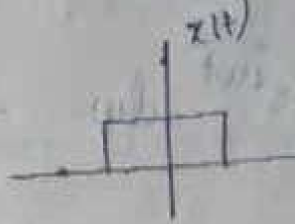
Fourier Transform

Fourier series is a tool to convert a signal from time domain to frequency domain but restricted to only priodic signal.

Fourier transform is an extension of Fourier series which is also made applicable for aperiodic signals

The basic idea behind the fourier transfor is with an assumption that an Aperiod signal is also period but with a time period '∞'.





$$x_p(t)\Big|_{\text{timeperiod}=T}$$

$$\lim_{T\to\infty} x_p(t) = x(t)$$

Applying fourier series for $x_p(t)$:

$$x_p(t) = \frac{1}{T}\sum_{K=-\infty}^{\infty} x(K)\cdot e^{jK\omega_0 t} = \frac{1}{T}\sum_{K=-\infty}^{\infty} x(K)\cdot e^{jK\omega_0 t}$$

$$x(K) = \frac{1}{T}\int_{\langle T\rangle} x_p(t)\, e^{-jK\omega_0 t}\, dt \quad \text{—— (1)}$$

As $T\to\infty$, the following changes takes place.

1. That is, the distance between frequency components decrease and at a point $T=\infty$, the Spectrum which was discrete turns continuous $\omega_0 = \frac{2\pi}{T} \downarrow\downarrow$

2. Since the spectrum turned continuous the term $K\cdot\omega_0$ as no meaning, we replace it by variable representing

frequency axis 'ω'.

3. $x(t) = \int_{-\infty}^{\infty} x(\omega) \cdot e^{j\omega t} \cdot d\omega$ —— (2)

The above expression is called synthesis equation which converts a signal from frequency domain to time domain.

From (1) $X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) e^{+j\omega t} \cdot dt$

$$x_p(t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(k) \cdot e^{jk\omega_0 t}$$

$$= \frac{\omega_0}{T} \int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t}$$

$$x(t) = \omega_0 \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} \cdot d\omega \rightarrow \text{synthesis equation}$$

from (1) $\Rightarrow$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \cdot dt \rightarrow \text{Analysis Equation}$$

which converts time domine to frequency domine

The term $e^{-j\omega t}$ is called as kernel

properties of kernel :-

1. It is a function of both variables time and frequency

2. Integrated with respect to t so that t gets eleminated resulting in a function of ω.

3. should be integrated from $-\infty$ to ∞, since an aperiod signal can lie any were on the time axis.

4. The forword and reverse kernels are always conjugate of each other.

Derichlet's conditions :-

The only condition for fourier transfor to exist is the signal must be absomlutly integrable

Fouries transform of some basic signals

1. $e^{-at} u(t)$

$$= \int_0^\infty e^{-at} \cdot dt$$

$$= \left[\frac{e^{-at}}{-a}\right]_0^\infty = 1/a$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} \cdot dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_{0}^{\infty}$$

$$= \frac{1}{a+j\omega}$$

Magnitude spectrum:-

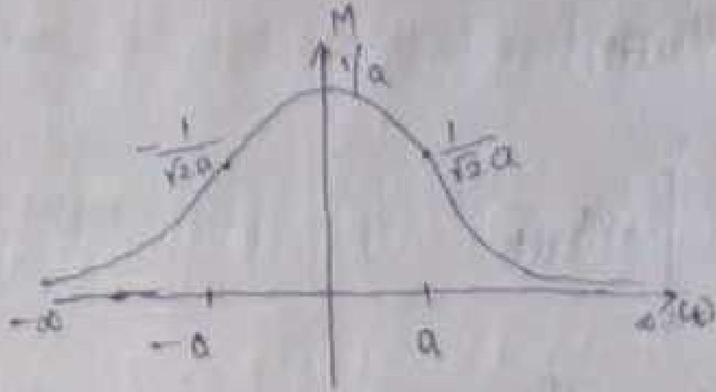
$$= \frac{1}{a+j\omega} \times \frac{a-j\omega}{a-j\omega}$$

$$X(\omega) = \frac{a-j\omega}{a^2+\omega^2}$$

$$M = \sqrt{\frac{a^2}{(a^2+\omega^2)^2} + \frac{\omega^2}{(a^2+\omega^2)^2}}$$

$$M = \frac{1}{\sqrt{a^2+\omega^2}}$$

ω	M
0	$1/a$
$\pm a$	$1/\sqrt{2}\,a$
$\pm\infty$	0

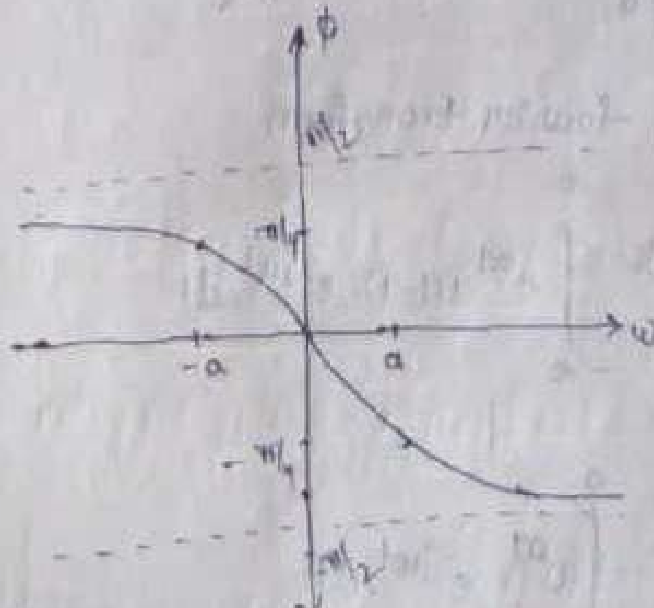


phase spectrum :-

$$\phi = -\tan^{-1}(b/a)$$

$$\phi = -\tan^{-1}(\omega/a)$$

ω	ϕ
$-\infty$	$\pi/2$
$-a$	$\pi/4$
0	0
a	$-\pi/4$
∞	$-\pi/2$



$\tan 90 = \pi/2$

$\tan 0 = 0$

$\tan 45 = 1$

2. $e^{at} u(-t)$

$$= \int_{-\infty}^{0} e^{at} \cdot dt$$

$$= \left[\frac{e^{at}}{a} \right]_{-\infty}^{0}$$

$$= \left[\frac{e^{0} - e^{-\infty}}{a} \right]$$

$$= 1/a$$

Apply fourier transform.

$$X(K) = \int_{-\infty}^{\infty} e^{at} u(-t) \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{-\infty}^{0} e^{at} \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t} \cdot dt$$

$$= \left[\frac{e^{(a-j\omega)t}}{a-j\omega} \right]_{-\infty}^{0} = X(K) = \frac{1}{a-j\omega}$$

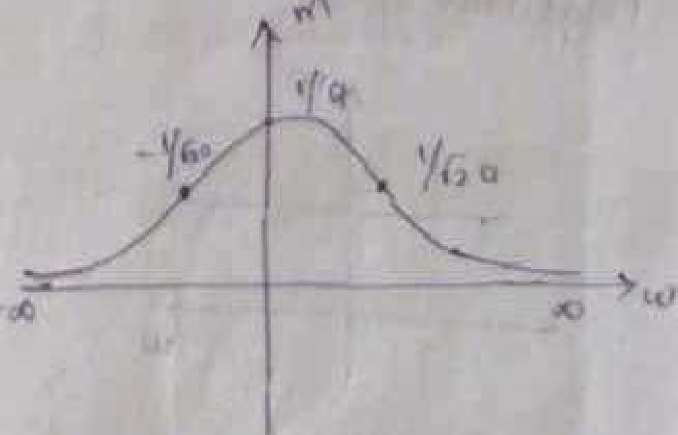
Magnitude spectrum:-

$$X(k) = \frac{1}{a-jw} \times \frac{a+jw}{a+jw}$$

$$X(k) = \frac{a+jw}{a^2+w^2}$$

$$M = \frac{1}{\sqrt{a^2+w^2}}$$

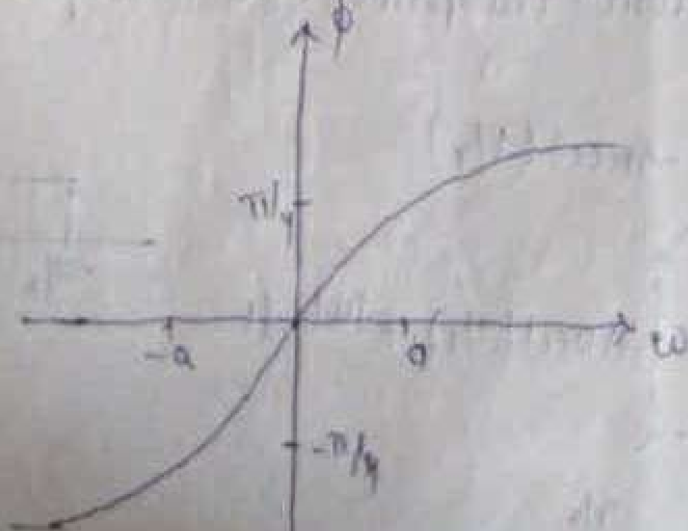
w	M
0	$1/a$
$\pm a$	$\frac{1}{\sqrt{2}a}$
$\pm\infty$	0



phase spectrum:-

$$\phi = \tan^{-1}(b/a)$$

w	ϕ
$-\infty$	$-\pi/2$
$-a$	$-\pi/4$
0	0
a	$\pi/4$
∞	$\pi/2$

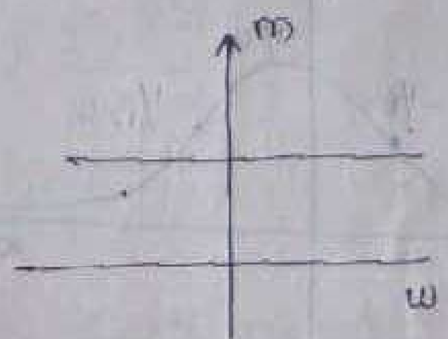


Fourier transform of $\delta(t)$:

$$= \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j\omega t} \cdot dt \quad \text{[from sifting property]}$$

$$= 1$$

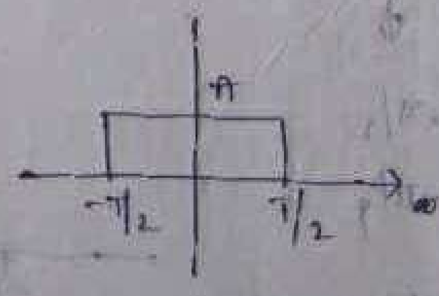
Magnitude spectrum.



For even signals phase spectrum does not exist.

Fourier trasformer of a rectangular signal :-

$A\,Rect(t/T)$



$$= \int_{-\infty}^{\infty} A\,Rect(t/T) \cdot e^{-j\omega t} \cdot dt$$

$$= A \int_{-T/2}^{T/2} e^{-j\omega t} \, dt$$

$$= \frac{A}{-j\omega}\left(e^{-j\omega t}\right)_{-T/2}^{T/2}$$

$$= \frac{A}{j\omega}\left[e^{j\omega T/2} - e^{-j\omega T/2}\right]$$

$$= \frac{2A}{\omega}\left[\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2j}\right]$$

$$= \frac{2A}{\omega}\sin(\omega T/2)$$

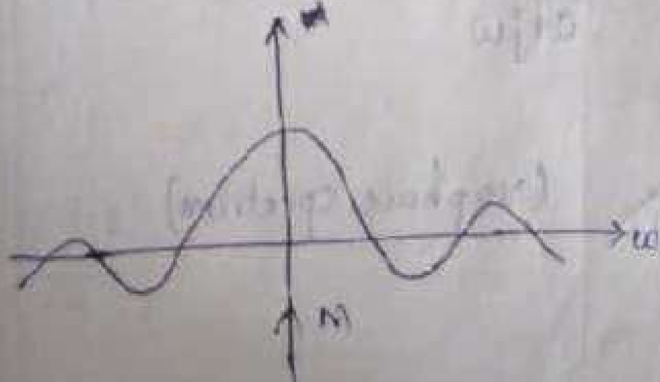
$$= A \cdot T \cdot \frac{\sin(\omega T/2)}{T\omega/2}$$

$$\boxed{\frac{\sin\theta}{\theta} = \text{sinc}}$$

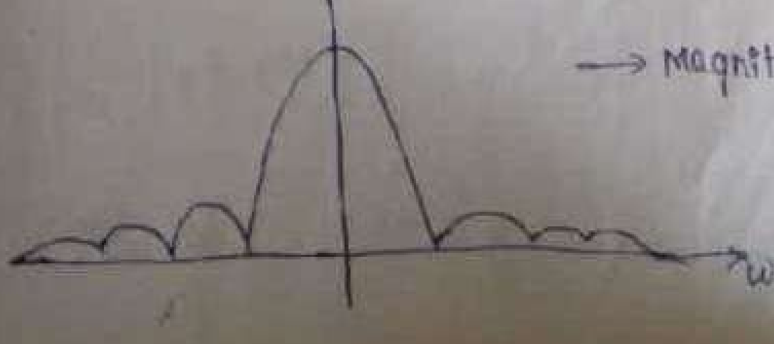
$$= AT \cdot \text{sinc}(\omega T/2)$$

$$x(\omega) = AT\text{Sa}(\omega T/2)$$

Spectrum for sampling function:-



be sampling f^n



→ Magnitude

Fourier transform of $e^{-a|t|}$

t is +ve $\rightarrow e^{-at}$

t is -ve $\rightarrow e^{at}$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-a|t|} u(t) \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{-\infty}^{\infty} [e^{at} u(-t) + e^{-at} u(t)] \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{-\infty}^{0} e^{at} \cdot e^{-j\omega t} \cdot dt + \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t} \cdot dt$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$X(\omega) = \frac{2a}{a^2+\omega^2}$$ (No phase spectrum)

Fourier transform of $e^{at}u(t)$.

$$= \int_{0}^{\infty} e^{at}.dt$$

$$= \left[\frac{e^{at}}{a}\right]_{0}^{\infty}$$

$$= \frac{1}{a}\left[e^{\infty} - e^{0}\right]$$

$$= \frac{1}{a}\left[\infty - 1\right]$$

$$= \infty$$

$$\int_{-\infty}^{\infty} e^{-st}.x(t).dt$$

$$\int_{0}^{\infty} e^{at}.e^{-st}.dt$$

$$\int_{0}^{\infty} e^{-(s-a)t}\,dt$$

$$\left(\frac{e^{-(s-a)t}}{-(s-a)}\right)_{0}^{\infty}$$

$$\boxed{\frac{1}{s-a}}$$

Fouries transform does not exict

$e^{-at}u(-t)$

$$= \int_{-\infty}^{0} e^{-at}.dt$$

$$= \left[\frac{e^{-at}}{-a}\right]_{-\infty}^{0}$$

$$= \frac{-1}{a}\left[e^{0} - e^{+\infty}\right]$$

$$= \cancel{-1/a}\ \infty \qquad \frac{-1+\infty}{a}$$

Properties of fourier transform :-

1. Linearity property :-

$$f_1(t) \longleftrightarrow F_1(\omega)$$

$$f_2(t) \longleftrightarrow F_2(\omega)$$

$$A f_1(t) + B f_2(t) \longleftrightarrow A F_1(\omega) + B F_2(\omega)$$

2. Time shifting property :

$$f(t) \longleftrightarrow F(\omega)$$

$$f(t-t_0) \longleftrightarrow e^{-j\omega t_0} F(\omega)$$

$$= \int_{-\infty}^{\infty} f(t-t_0) \cdot e^{-j\omega_0 t} \cdot dt$$

$$t - t_0 = \tau$$

$$t = \tau + t_0$$

$$= \int_{-\infty}^{\infty} f(\tau) \cdot e^{-j\omega_0(\tau + t_0)} \cdot d\tau$$

$$= \int_{-\infty}^{\infty} f(\tau) \cdot e^{-j\omega_0 \tau} \cdot e^{-j\omega t_0} \cdot d\tau$$

$F(\omega) = e^{-j\omega t_0} \cdot F(\omega)$

Time scaling property:

$$f(t) \longleftrightarrow F(\omega)$$

$$f(at) \longleftrightarrow F(\omega) \frac{1}{|a|} \cdot F(\omega/a)$$

$$F(\omega) = \int_{-\infty}^{\infty} f(at) \cdot e^{-j\omega t} \cdot dt$$

a is +ve:- $at = \tau$

$$dt = \frac{1}{a} \cdot d\tau$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} f(\tau) \cdot e^{-j\omega \tau/a} \, d\tau$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} f(\tau) \cdot e^{-j(\omega/a)\tau} \, d\tau$$

$$= \frac{1}{a} \cdot F(\omega/a)$$

a is -ve:- $-at = \tau$

$$dt = -d\tau/a$$

$$= \frac{-1}{a} \int_{\infty}^{-\infty} f(\tau) \cdot e^{-j(-\omega/a)\tau} \, d\tau$$

$$= \frac{1}{a}\int_{\infty}^{-\infty} f(\tau)\cdot e^{-j(-\omega/a)\tau}\, d\tau$$

$$F(\omega) = \frac{1}{a} F(-\omega/a)$$

Time reversal :-

Substituting $a = -1$ in scaling property

$$f(at) \longleftrightarrow \frac{1}{|a|} F(\omega/a)$$

$$F(-t) \longleftrightarrow F(-\omega)$$

Differentiation in time domain :-

$$f(t) \longleftrightarrow F(\omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

Synthesis eq

$$\frac{d}{dt}(f(t)) \longleftrightarrow j\omega\, F(\omega)$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\cdot e^{j\omega t}\, d\omega$$

$$\frac{d}{dt}(f(t)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\cdot j\omega \cdot e^{j\omega t}\, d\omega$$

$$\frac{d}{dt}(f(t)) = F^{-1}\{F(\omega), j\omega\}$$

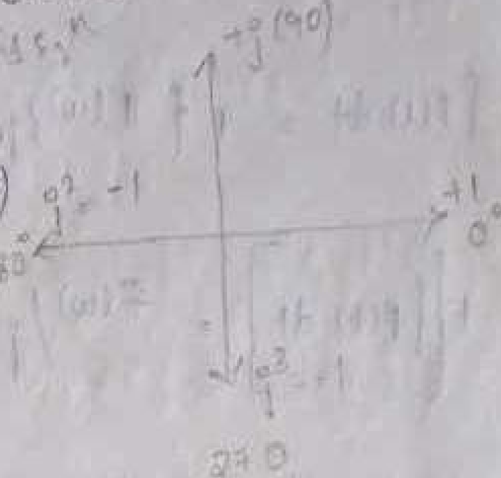
$$= F\left\{\frac{d}{dt}(f(t))\right\} = j\omega \cdot F(\omega)$$

Differentiation in frequency domain :-

Analysis eqn

$$f(t) \longleftrightarrow F(\omega)$$

$$t \cdot f(t) \longleftrightarrow j \frac{d}{d\omega} F(\omega)$$



$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} \, dt$$

$$\frac{d}{d\omega}(F(\omega)) = \int_{-\infty}^{\infty} f(t)(-jt) \cdot e^{-j\omega t} \, dt$$

$j^1 = \sqrt{-1} = +j$

$j^2 = (\sqrt{-1})^2 = -1$

$j^3 = (\sqrt{-1})^3 = -j$

$$\frac{d}{d\omega} F(\omega) = F\{(-jt) \cdot f(t)\}$$

$j^4 = (\sqrt{-1})^4 = +1$

$$(-j) \frac{-1}{j} \frac{d}{d\omega} F(\omega) = F\{t \cdot f(t)\}$$

$$\frac{1}{j} = -j = e^{-j\pi/2}$$

Integration in time domain :-

Synthesis

$$f(t) \longleftrightarrow F(\omega)$$

$$\int f(t) \cdot dt \longleftrightarrow \frac{1}{j\omega} F(\omega)$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \, d\omega$$

$$\int f(t) \cdot dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \cdot \frac{e^{j\omega t}}{j\omega} \cdot d\omega$$

$$\int f(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\cdot\frac{1}{j\omega}\, e^{j\omega t}\,d\omega$$

$$\int f(t)\,dt = F^{-1}\left\{ F(\omega)/j\omega \right\}$$

$$F\left[\int f(t)\cdot dt\right] = F(\omega)/j\omega$$

Integration in frequency domain.

$$f(t) \longleftrightarrow F(\omega)$$

$$\overset{f(t)/t}{\int F(\omega)\,d\omega} \longleftrightarrow F^{-1}\int F(\omega)\cdot d\omega$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\cdot dt$$

$$\int F(\omega)\cdot d\omega = \int_{-\infty}^{\infty} f(t)\cdot\frac{e^{-j\omega t}}{-jt}\cdot dt$$

$$\int F(\omega)\cdot d\omega = F\left[\frac{f(t)}{-jt}\right]$$

frequency shifting property (or) modulation property:-

$$f(t) \longleftrightarrow F(\omega)$$

$$e^{j\omega_0 t} f(t) \longleftrightarrow F(\omega-\omega_0)$$

$$= \int_{-\infty}^{\infty} e^{j\omega_0 t} \cdot f(t) \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{-\infty}^{\infty} f(t) \cdot e^{-j(\omega-\omega_0)t} \cdot dt$$

$$= F(\omega-\omega_0)$$

Application of properties :-

1. find the fourier transform of $t \cdot e^{-at} u(t)$

$$f(t) = t \cdot e^{-at} u(t).$$

$$t \cdot e^{-at} u(t) \longleftrightarrow j \cdot \frac{d}{d\omega}\left(\frac{1}{a+j\omega}\right)$$

$$t \cdot f(t) \longleftrightarrow j \frac{d}{d\omega}(F\omega)$$

$$j \frac{d}{d\omega}\left(\frac{1}{a+j\omega}\right) = -1 \cdot j^2 \frac{1}{(a+j\omega)^2}$$

$$= \frac{1}{(a+j\omega)^2}$$

2. $f(t) = e^{-at} u(t)$

$$F\{f(t-4)\} = \frac{e^{-4j\omega}}{a+j\omega}$$

3. $\delta(t-4)$

$\delta(t-4) \longleftrightarrow e^{-4j\omega}(1)$ (According to time shifting property)

** Duality property :-

$$f(t) \longleftrightarrow F(\omega)$$

$$F(t) \longleftrightarrow 2\pi f(-\omega)$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \cdot e^{j\omega t} \cdot d\omega.$$

$$2\pi f(t) = \int_{-\infty}^{\infty} F(\omega) \cdot e^{j\omega t} \cdot d\omega$$

$t = -t$

$$2\pi f(-t) = \int_{-\infty}^{\infty} F(\omega) \cdot e^{-j\omega t} \cdot d\omega$$

$\omega \Longleftrightarrow t$

$$2\pi f(-\omega) = \int_{-\infty}^{\infty} F(t) \cdot e^{-j\omega t} \cdot dt$$

$$2\pi f(-\omega) = F\{F(t)\}$$

Example :-

find the fourier transform of $\frac{1}{a+jt}$

$$e^{-at} u(t) \longleftrightarrow \frac{1}{a+j\omega}$$

from Duality property

$$\frac{1}{a+jt} \longleftrightarrow 2\pi e^{a\omega} u(-\omega)$$ (t is replaced with $-\omega$)

find the fourier transform of $\frac{1}{1+t^2}$

$$e^{-a|t|} = e^{-at} u(t) + e^{at} u(-t)$$

$$e^{-a|t|} \longleftrightarrow \frac{2a}{a^2+\omega^2}$$

$$\frac{1}{2} e^{-a|t|} \longleftrightarrow \frac{a}{a^2+\omega^2}$$

$a = 1$

$$\frac{1}{1+t^2} \longleftrightarrow \frac{1}{2}(2\pi) \cdot e^{-a|\omega|} = \pi \cdot e^{-|\omega|}$$

Q: find the fourier transform of 1

$\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\,\delta(\omega)$

$2\pi\,\delta(\omega)$ (Impulse lies at origin)

Q: find the inverse fourier transform of $\frac{e^{-2j\omega}}{1+j\omega}$

from time shifting property $f(t-t_0) \longleftrightarrow e^{-j\omega t_0} F(\omega)$

$t_0 = 2$

$e^{-at}u(t) \longleftrightarrow \frac{a}{a+j\omega}$

$e^{-t}u(t) \longleftrightarrow \frac{1}{1+j\omega}$ $\longrightarrow$ since $f(t) = e^{-t}u(t)$

$\therefore\ f(t-2) \longleftrightarrow e^{-(t-2)}.u(t-2)$

~~Since~~

find the fourier transform of $e^{j\omega_0 t}$

$t \longrightarrow t_0$

$\omega \rightarrow \omega_0$

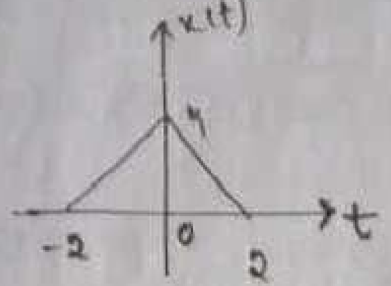
$e^{j\omega_0 t} \longleftrightarrow F(\omega-\omega_0)$

$e^{j\omega_0 t}(1) \longleftrightarrow 2\pi\delta(\omega-\omega_0)$ (frequency shifting property)

$1 \xleftrightarrow{FT} 2\pi\delta(\omega)$

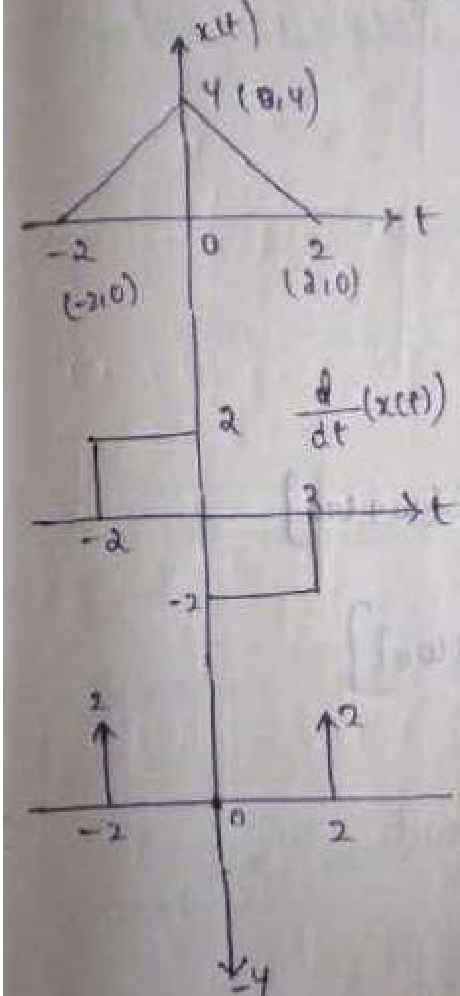
$2\pi\delta(\omega-\omega_0)$

find the fourier transform of



$2t+4 \ (-2 \text{ to } 0), \frac{d}{dt}(2t+4)=2$

$-2t+4 \ (0 \text{ to } 2), \frac{d}{dt}(-2t+4)=-2$



$\frac{d}{dt}(f(t)) \longleftrightarrow jw F(w)$

$\frac{d^2}{dt^2} f(t) \longleftrightarrow (jw)(jw)F(w)$

$$\frac{d^2}{dt^2}(x(t)) = 2\delta(t+2) - 4\delta(t) + 2\delta(t-2)$$

$$(jw)^2 \cdot F(w) = 2e^{2jw}\times 1 - 4\times 1 + 2e^{-2jw}\times 1$$

$$-w^2 \cdot F(w) = -4 + 4\left[\frac{e^{2jw}+e^{-2jw}}{2}\right]$$

$$F(\omega) = \frac{4\cos 2\omega - 4}{-\omega^2}$$

$$F(\omega) = \frac{4 - 4\cos 2\omega}{\omega^2}$$

find the fourier transform of $\cos\omega_0 t$

$$\cos\omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$= \frac{1}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]$$

$$= \frac{1}{2}\left[2\pi\delta(\omega - \omega_0) + 2\pi\delta(\omega + \omega_0)\right]$$

$$\cos\omega_0 t = \pi\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]$$

find the fourier transform of $\sin\omega_0 t$

$$\sin\omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

$$= \frac{1}{2j}\left[e^{j\omega_0 t} - e^{-j\omega_0 t}\right]$$

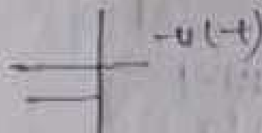
$$= \frac{1}{2j}\left[2\pi\delta(\omega - \omega_0) - 2\pi\delta(\omega + \omega_0)\right]$$

$$\sin\omega_0 t = \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

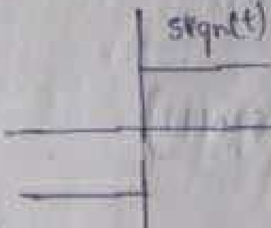
Signum function :- (sgn)

$sgn(t) = u(t) \quad ; t > 0$

$sgn(t) = -u(-t) \quad ; t < 0$



$$sgn(t) = u(t) + [-u(-t)]$$
$$= u(t) - u(-t)$$



find the fourier transform of signum function.

$$Sgn(t) = e^{-at}u(t) - e^{at}u(-t)$$

$$\underset{a\to 0}{Lt}\left[e^{-at}u(t) - e^{at}u(-t)\right] = u(t) - u(-t) = sgn(t)$$

$$F\{sgn(t)\} = \underset{a\to 0}{Lt}\left[\frac{1}{a+j\omega} - \frac{1}{a-j\omega}\right]$$

$$= \underset{a\to 0}{Lt}\left[\frac{a-j\omega-a-j\omega}{a^2+\omega^2}\right]$$

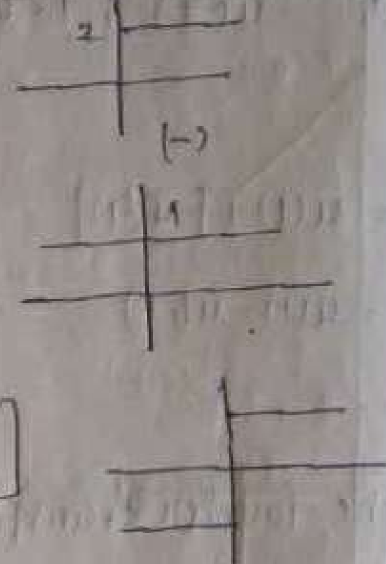
$$= \underset{a\to 0}{Lt}\left[\frac{-2j\omega}{a^2+\omega^2}\right]$$

$$= \frac{-2j\omega}{\omega^2}$$

$$sgn(t) = \frac{2}{j\omega}$$

Find the fourier transform of step signal.

$$sgn(t) = 2u(t) - 1$$



$$sgn(t) + 1 = 2u(t)$$

$$u(t) = \frac{1}{2}\left[1 + sgn(t)\right]$$

$$F\{u(t)\} = \frac{1}{2}\left[2\pi\,\delta(\omega) + \frac{2}{j\omega}\right]$$

$$F\{u(t)\} = \pi\,\delta(\omega) + \frac{1}{j\omega}$$

find the fourier transform of $1/\pi t$

$$\left.\begin{array}{l} f(t) \longleftrightarrow F(\omega) \\ F(t) \longleftrightarrow 2\pi f(-\omega) \end{array}\right\} \text{duality property}$$

$$sgn(t) \longleftrightarrow \frac{2}{j\omega}$$

$$j \cdot \frac{1}{2} \cdot sgn(t) \longleftrightarrow \frac{1}{\omega}$$

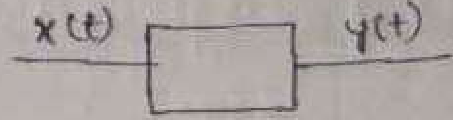
$$\frac{1}{t} \longleftrightarrow 2\pi \cdot j \cdot \frac{1}{2} \cdot sgn(-\omega)$$

$$\frac{1}{t} \longleftrightarrow -\pi \cdot \frac{1}{j} \cdot \operatorname{sgn}(\omega) \qquad \operatorname{sgn}(-\omega) = -\operatorname{sgn}(\omega)$$

$$\frac{1}{\pi t} \longleftrightarrow -\frac{1}{j} \operatorname{sgn}(\omega)$$

System and its properties

System: A physical device which produces desired output with aviliable input



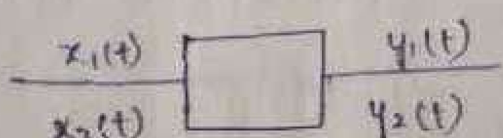
$$y(t) = T[x(t)]$$

Where, T is an operator.

Properties of systems :-

1. Linear and non-linear systems :-

A system is said to be linear if it satisfies homoginity and superposition theorem.



$$a x_1(t) + a x_2(t) = a y_1(t) + a y_2(t)$$

Test for linearity of a system:

1. $y(t) = 2x(t)$

$x_1(t) \longrightarrow y_1(t) = 2x_1(t)$

$x_2(t) \longrightarrow y_2(t) = 2x_2(t)$

$x_1(t) + x_2(t) \longrightarrow y_3(t) = 2[x_1(t) + x_2(t)]$

$y_3(t) = y_1(t) + y_2(t)$ then the system is linear

$y_3(t) \neq y_1(t) + y_2(t)$ then the system is non-linear

2. $y(t) = x^2(t)$

$x_1(t) \longrightarrow y_1(t) = x_1^2(t)$

$x_2(t) \longrightarrow y_2(t) = x_2^2(t)$

$x_1(t) + x_2(t) \longrightarrow y_3(t) = [x_1(t) + x_2(t)]^2$

$y_3(t) \neq y_1(t) + y_2(t)$

$\therefore$ System is Non-linear.

3. $y(t) = e^{x(t)}$

$x_1(t) \longrightarrow y_1(t) = e^{x_1(t)}$

$x_2(t) \longrightarrow y_2(t) = e^{x_2(t)}$

$x_1(t) + x_2(t) \longrightarrow y_3(t) = [e^{x_1(t)} + e^{x_2(t)}] \quad e^{x_1(t) + x_2(t)}$

$y_3(t) \neq y_1(t) + y_2(t)$

4. $y(t) = 3x(t) + 4$

$x_1(t) \longrightarrow y_1(t) = 3x_1(t) + 4$

$x_2(t) \longrightarrow y_2(t) = 3x_2(t) + 4$

$x_1(t) + x_2(t) \longrightarrow y_3(t) = [\cancel{(3x_1(t)+4) + (3x_2(t)+4)}]$

$= [3x_1(t) + 3x_2(t)] + 4$

$y_3(t) = 3[x_1(t) + x_2(t)] + 4$

$y_3(t) \neq y_1(t) + y_2(t+2)$ (Non-linear)

5. $y(t) = x(t^2)$

$x_1(t) - y_1(t) = x_1(t^2)$

$x_2(t) - y_2(t) = x_2(t^2)$

$x_1(t) + x_2(t) \longrightarrow y_3(t) = x_1(t^2) + x_2(t^2)$

$y_3(t) = y_1(t) + y_2(t)$

Linear system.

6. $y(t) = \sqrt{x(t)}$

$y(t) = \log[x(t)]$

this are non-linear systems

2. Time variant and Time invariant system :-

(or)

Shift variant and shift invariant

A system is said to be time invariant if the input and output relation doesn't change with time.

Test for time invariance :-

1. $y(t) = 2x(t)$

$x(t) \longleftarrow y(t)$

Shift only input by t_0 $\quad y(t) = 2x(t-t_0)$

Shift the entire relation by t_0 $\quad y(t-t_0) = 2x(t-t_0)$

Both the inputs are same, Time invariant

2. $y(t) = t\,x(t)$

Shift only input by t_0 = $y(t) = t\,x(t-t_0)$

$y(t-t_0) = (t-t_0)\,x(t-t_0)$

3. $y(t) = x(t^2)$

$y(t) = x((t-t_0)^2)$

$y(t) = x(t^2 - t_0)$

$y(t-t_0) = x[(t-t_0)^2]$

$\therefore$ The $y(t) \neq y(t-t_0)$, Time variant

4. $y(t) = x(-t)$

$y(t) = x(-t-t_0)$

$y(t-t_0) = x(-t+t_0)$

$y(t) \neq y(t-t_0)$, Time variant

5. $y(t) = x(2t)$

$y(t) = x(2t-t_0)$

$y(t-t_0) = x(2(t-t_0))$

$y(t) \neq y(t-t_0)$, Time variant

6. $y = x^2(t)$

$y(t) = x^2(t-t_0)$

$y(t-t_0) = x^2(t-t_0)$

$y(t) = y(t-t_0)$, time invariance and linear system

3. Static and dynamic systems :-

A system is said to be static if the present output depends only on the present input, hence this system doesn't need Memory.

For a dynamic system, the system depends on atleast one past or future alon·values along with the present input. Hence this system needs memory.

Test for S'

A system is static as long as the 't' in $x(t)$ and $y(t)$ are exactly same. Any change makes the system dynamic.

Ex :-

Same

$y(t) = 2x(t)$ - static

diff

$y(t) = 2x(2t)$ = dynamic.

$y(t) = t\,x(t)$ - Static

$y(t) = x(\cos t)$ dynamic

$y(t) = \cos t\, x(t)$ Static

4. Causal and Non-causal system :-

A system is said to be causal if the output depends only on present and past values but not on future values whereas the output of a non-causal system depends on atleast one future value.

example :

1. $y(t) = x(-t)$

$t=1 \quad y(1) = x(-1)$

$t=-1 \quad y(-1) = x(1)$ } Non causal

2. $y(t) = x(t+1)$ Non causal

3. $y(t) = ax(t-1)$ Causal

4. $y(t) = \cos x(t)$ causal

5. $y(t) = \cos[x(t)]$ Causal

6. $y(t) = x[\cos t]$

$y(-\pi/4) = x(0.7)$ Non causal

All static systems are causal

statement - I :

All static systems are causal (T)

statement - II :

All causal systems are static (F) $y(t) = 3x(t-1)$ dynamic

statement - III :

All dynamic systems are non-causal (F) $y(t) = 2x(t-1)$ causal

statement - IV :

All non-causal systems are dynamic (T)

statement - V :

All causal systems are dynamic (F) $y(t) = x(t+1)$ non causal

statement - VI :

All static systems are non-causal (F) $y(t) = \cos[x(t)]$ causal.

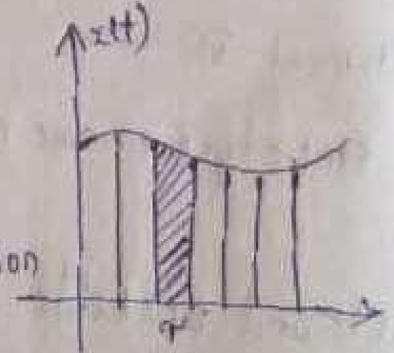
Stable and unstable systems:

A system is said to be stable if it produces a bounded output for a bounded input. Such systems are said to be BIBO stable; here bounded means finite

Impulse response :

The response of a system for an impulse signal as an input is called as impulse response. It is denoted by $h(t)$.

Any arbitary signal $x(t)$ can be represented as a running sum of impulses as shown



since, an impulse doesn't exist we divide the signal $x(t)$ into rectangular boxes, each of duration of τ. We start compressing it till $\tau = 0$, forming an impulse. The response of any System can be obtained by collecting all the responses for impulses at different positions. The response so Collected are impulse responses denoted by $h(t)$.

The overall response is then given by

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau$$

$$y(t) = x(t) * h(t)$$

where '*' represents the convolution operation.

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = h(t) * x(t)$$

Note:

1. convolution satisfies cummutative, Associative and distributive properties

$$x(t) * h_1(t) = h_1(t) * x(t)$$

$$x(t) * (h_1(t) * h_2(t)) = (x(t) * h_1(t)) * h_2(t)$$

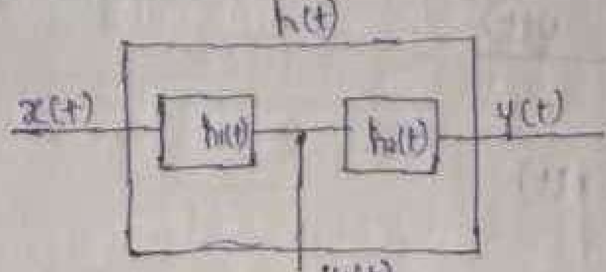
$$x(t) * [h_1(t) + h_2(t)] = [x(t) * h_1(t)] + [x(t) * h_2(t)].$$

2. The system should be an LTI system for evaluating response through impulse response linear and time invariant

Inter connection of systems :-

1. cascade :

Let two systems with individual impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade (series)



$$y(t) = x(t) * h(t) \quad — (1)$$

$$y(t) = y_1(t) * h_2(t)$$

But $y_1(t) = x(t) * h_1(t)$

$$= x(t) * h_1(t) * h_2(t)$$

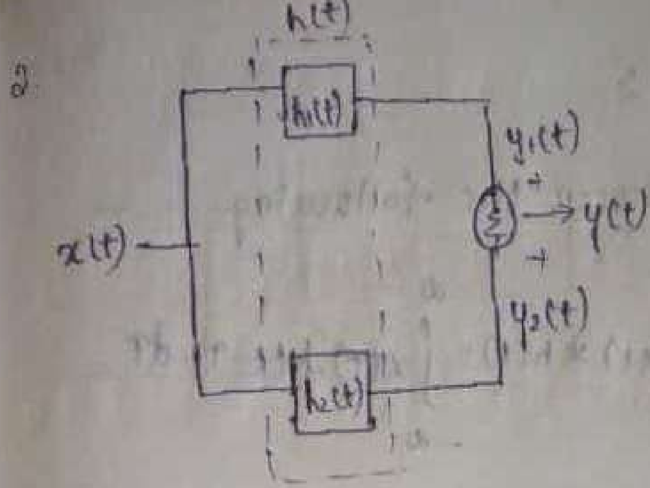
$$y_1(t) = x(t) * [h_1(t) * h_2(t)] \quad — (2)$$

$$\boxed{h(t) = h_1(t) * h_2(t)}$$

Therefore the overall impulse response is equal to

$$y(t) = [h_1(t) * h_2(t)] . x(t)$$

2



$$y(t) = y_1(t) + y_2(t)$$

$$y_1(t) = \left[x(t) * h_1(t)\right] + \left[x(t) * h_2(t)\right]$$

$$y_1(t) = x(t) * \left[h_1(t) + h_2(t)\right]$$

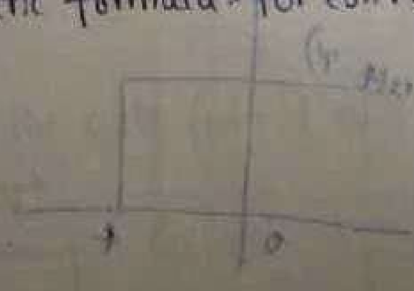
by comparing the above eq^n with $y(t) = x(t) * h(t)$

$$\boxed{h(t) = h_1(t) + h_2(t)}$$

Methods for finding convolution :-

Their are 3 ways to find convolution

1. Graphical Method
2. using fourier transforms
3. using the formula for convolution.



1. Graphical convolution :-

Find the convolution between the following.

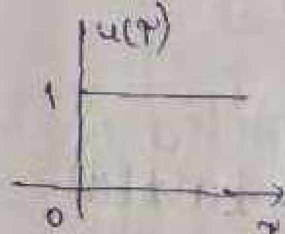
1. $u(t)$ and $e^{-t}u(t)$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau).h(t-\tau).d\tau$$

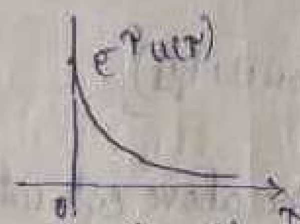
$u(t) \longleftrightarrow$

Step-I : change axis from t to τ

$u(\tau)$, $e^{-\tau}u(\tau)$



$h(t-\tau)$



$(x(\tau))$

Step-II : Evaluate the limits, sum of lower limits to sum of upper limits

$u(\tau)$, $e^{-\tau}u(\tau)$

0 to ∞ 0 to ∞

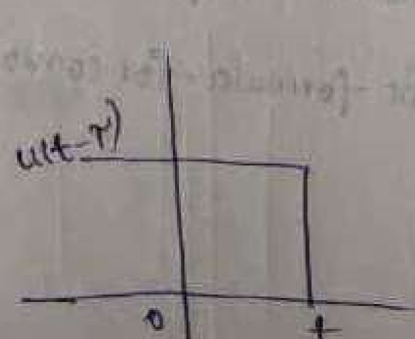
Step-III : Fold and shift of the two signals

$0 \le \tau \le \infty$

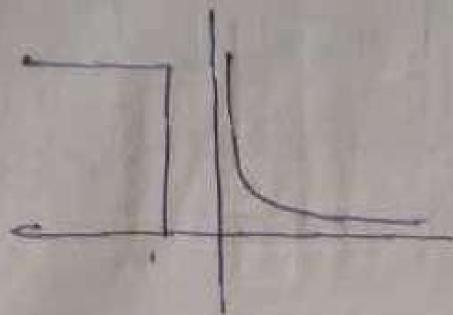
$0 \le t-\tau \le \infty$

$-t \le -\tau \le \infty$

$-\infty \le \tau \le t$

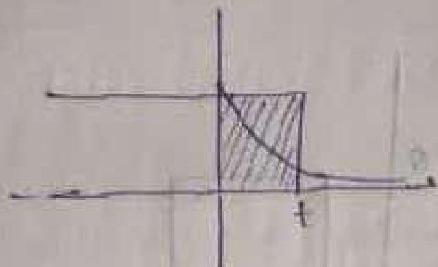


case-I: $t<0$



$$\int e^{-\tau} u(t-\tau)\, d\tau = 0$$

case-2: $t \geq 0$

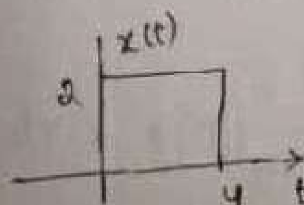


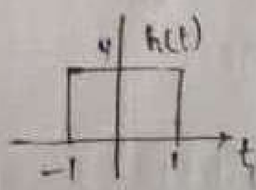
$$= \int_0^t e^{-\tau} \cdot 1\, d\tau$$

$$= \left[\frac{e^{-\tau}}{-1}\right]_0^t$$

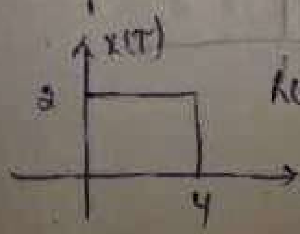
$$= 1 - e^{-t}$$

2. Find the convolution between

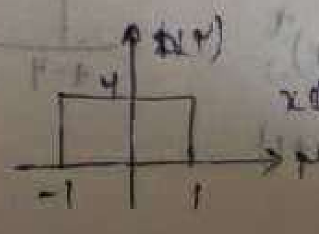




1. change the axis from t to τ



$h(t-\tau)$



$x(t)$

2. Evaluate the limits,

$x(\tau)$ $\quad$ $h(\tau)$

0 to 4 $\quad$ -1 to 1

= -1 to 5

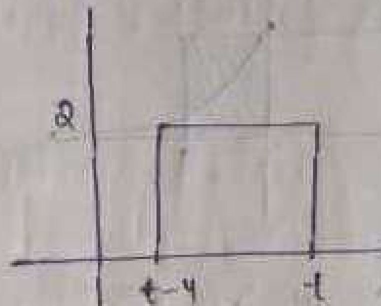
3. Fold and shift of the two signals.

case-I: $0 \le \tau \le 4$

$0 \le t-\tau \le 4$

$-t \le -\tau \le 4-t$

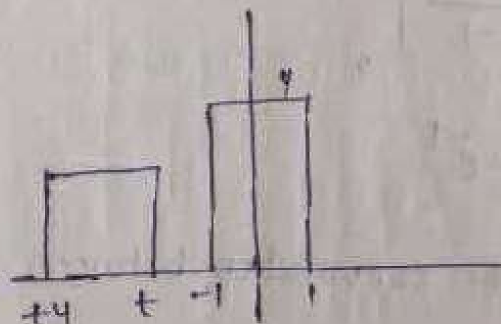
$t-4 \le \tau \le t$



case-I:

$t < -1$

Convolution = 0



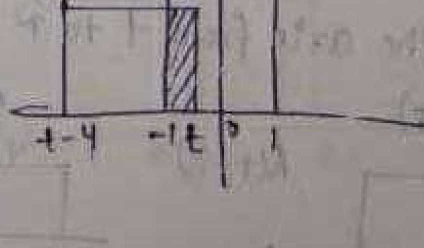
case-II:

$t > -1$ and $t-4 < -1$

$t < 3$

$$\int_{-1}^{t} 8\, d\tau = 8(\tau)_{-1}^{t}$$

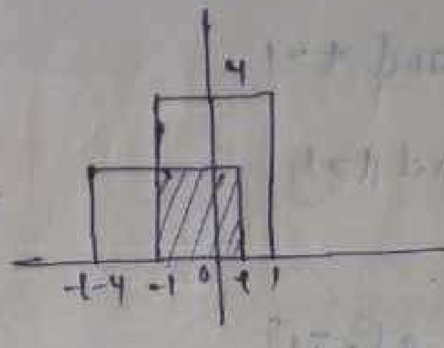
$$= 8(t+1)$$



case-III :

$t-4 < -1$ and $t > 0$

$t < 3$

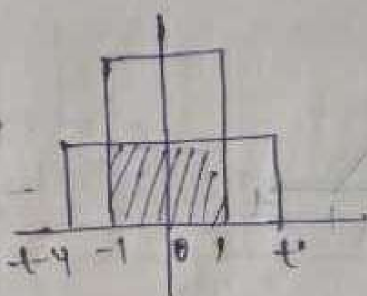


$$\int_{-1}^{t} 8 \cdot d\tau = 8(t+1)$$

Case-IV :

$t-4 < -1$ and $t > 1$

$t < 3$



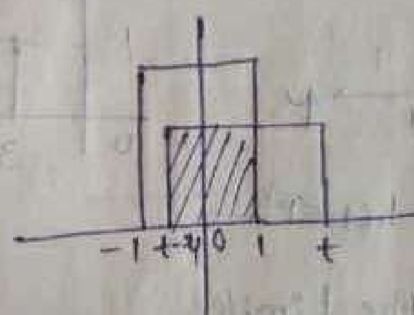
$$\int_{-1}^{1} 8 \cdot d\tau = 8[1+1]$$

$$= 16$$

case-V :

$t-4 > -1$ and $t > 1$

$t > 3$ and $t > 1$



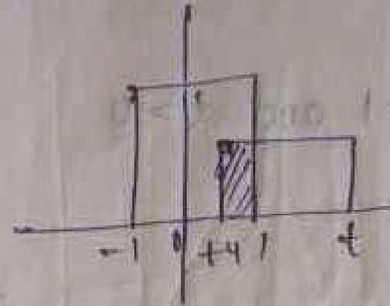
$$\int_{t-4}^{1} 8 \cdot d\tau = 8[\tau]_{t-4}^{1}$$

$$= 8[1-t+4]$$

$$= 8[5-t]$$

Case-VI:

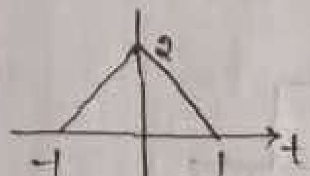
$t-4<1$ and $t>1$

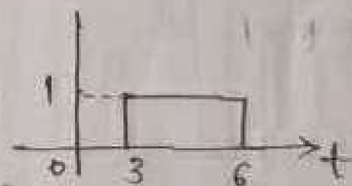


$t<5$ and $t>1$

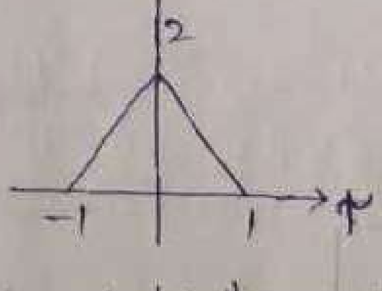
$$\int_{t-4}^{1} 8 \cdot d\tau = 8[5-t]$$

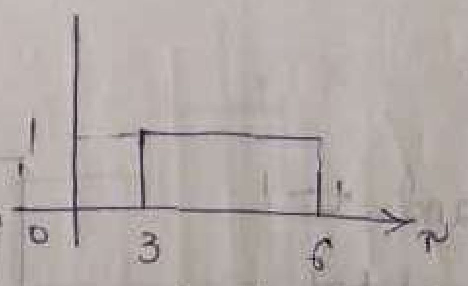
3. find the convolution between





1. change the Axis from t to τ





$x(\tau)$ ~~h(-τ)~~ h(~~t-τ~~) ~~h(-τ)~~ $h(t-\tau)$

2. Evaluate the limits

$x(\tau)$: -1 to 1 $h(t-\tau)$ = 3 to 6

2 to 7

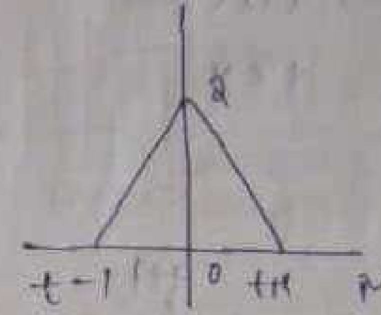
3. fold and shift of the signal

$-1 \le \tau \le 1$

$-1 \le t-\tau \le 1$

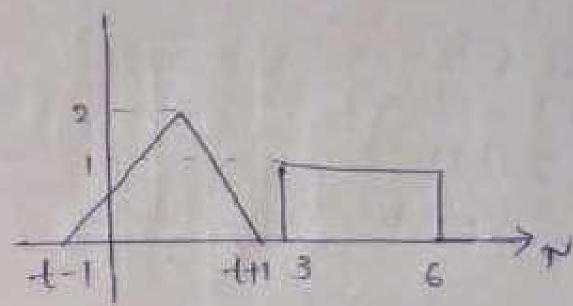
$(-1-t) \le -\tau \le 1-t$

$t-1 \le \tau \le t+1$

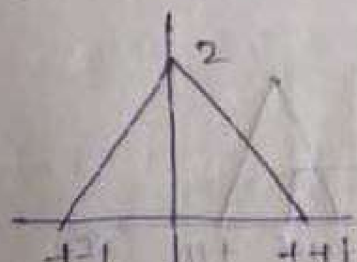


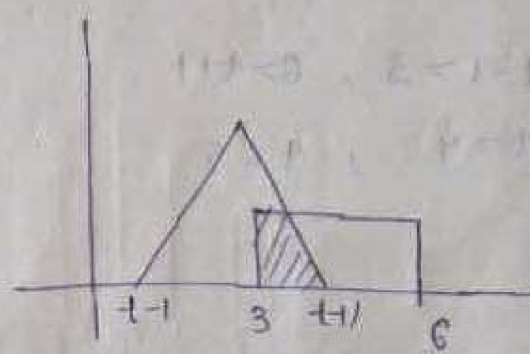
4. case-I

convolution = 0



Case-II:





$x(\tau) = 2\tau+2 \quad (-1 \text{ to } 0)$

$= -2\tau+2 \quad (0 \text{ to } 1)$

$t-1<3 \Rightarrow t<4$

$t+1>3 \Rightarrow t>2$

$x(t-\tau)$

$2\tau+2 \Rightarrow 2(t-\tau)+2$

$2t-2\tau+2$

$-2\tau+2 \Rightarrow -2t+2\tau+2$

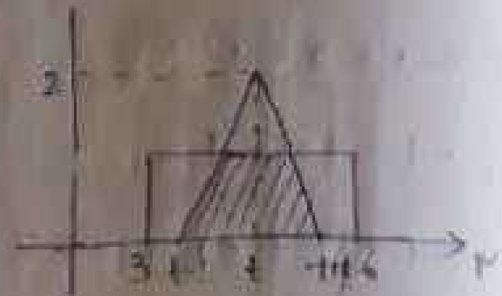
$= \int_{3}^{t+1} (-2t+2\tau+2)\,d\tau$

$=$

Case-III:

$t-1>3 \;,\; t+1<6$

$t>4 \qquad t<4$



$$\int_{t-1}^{t} (2t-2\tau+2)\,d\tau + \int_{t}^{t+1} (-2t+2\tau+2)\,d\tau$$

$$\left[2t\tau - \frac{2\tau^2}{2} + 2\tau\right]_{t-1}^{t} + \left[-2t\tau + \frac{2\tau^2}{2} + 2\tau\right]_{t}^{t+1}$$

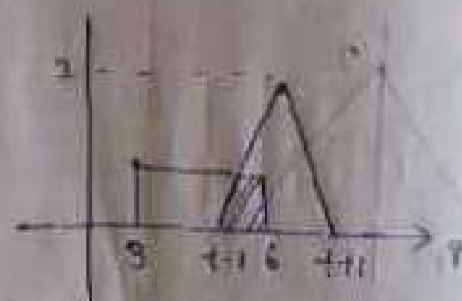
$$= 2t(t-t+1) - [t^2-(t-1)^2] + 2[t-t+1] + \{-2t(t+1-t) + (t+1)^2$$
$$-t^2 + 2(t+1-t)$$

$= 3+3 \Rightarrow 6$

Case-IV:

$t-1>3 \;,\; 6>t+1$

$t>4 \;,\; t<5$



$$\int_{t-1}^{6} 1\cdot(2t-2\tau+2)\,d\tau = \left[2t\tau - \frac{2\tau^2}{2} + 2\tau\right]_{t-1}^{6}$$

$$= 2t(6-t+1) - [6^2-(t-1)^2] + 2[6-t+1]$$

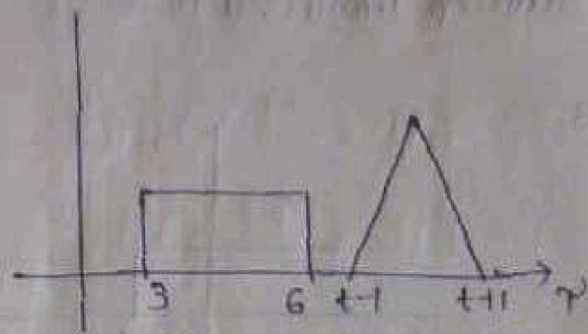
$$= 2t(7-t) - 36 + t^2 + 1 - 2t + 2[7-t]$$

$$= 14t - 2t^2 - 35 + t^2 - 2t + 14 - 2t$$

$$= -t^2 + 10t - 22$$

case-V:

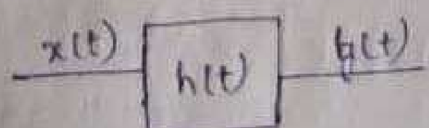
convolution is zero



find the response of the system with impulse response $h(t) = u(t-2)$, when excited by inputs $x(t) = u(t+1)$

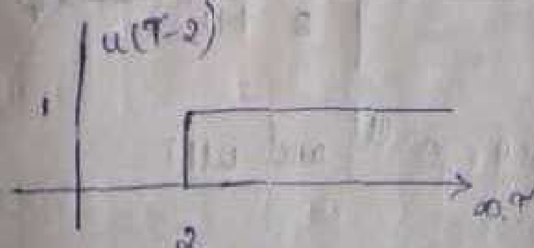
$h(t) = u(t-2)$

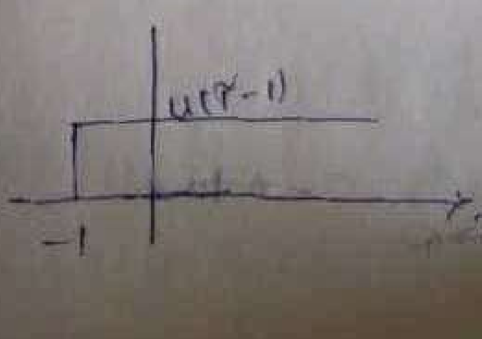
$x(t) = u(t-1)$



$y(t) = x(t) * h(t)$

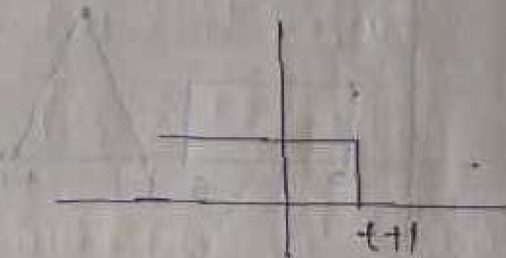
1.





2. Sum of limits : 1 to ∞

3.



$-1 \le \tau \le \infty$

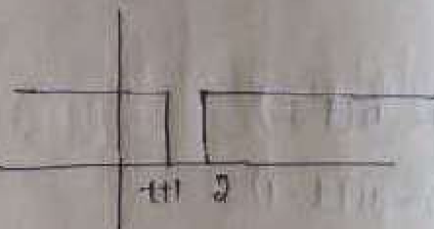
$-1 \le t-\tau \le \infty$

$-t-1 \le -\tau \le \infty$

$-\infty \le \tau \le t+1$

case-I :

convolution is zero



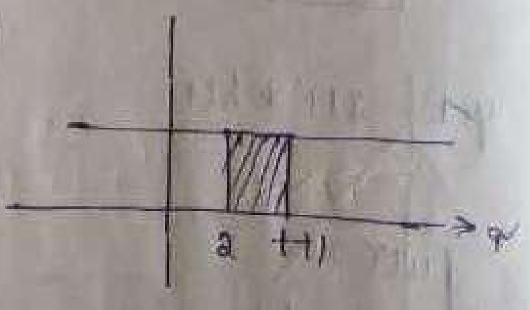
$t+1 < 2$

$t < 1$

case-II :

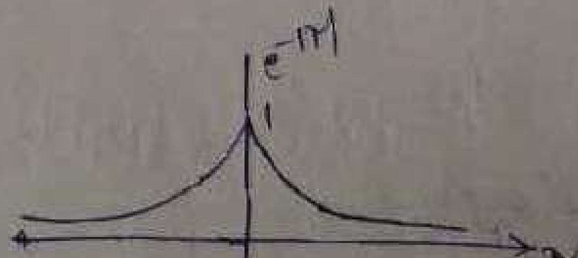
$t+1 > 2 \Rightarrow t > 1$

$$\int_{2}^{t+1} 1 \cdot d\tau = t-1$$



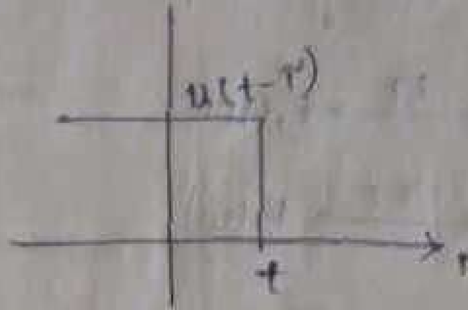
find the convolution between $e^{-|t|}$ and $u(t)$

$e^{-|t|} = e^{-t}u(t) + e^{t}u(-t)$



$-\infty$ to ∞

$0 \le \tau \le \infty$



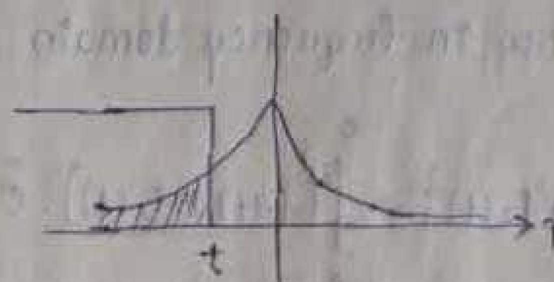
$0 \le t-\tau \le \infty$

$-t \le -\tau \le \infty$

$-\infty \le \tau \le t$

Case-I : $t < 0$

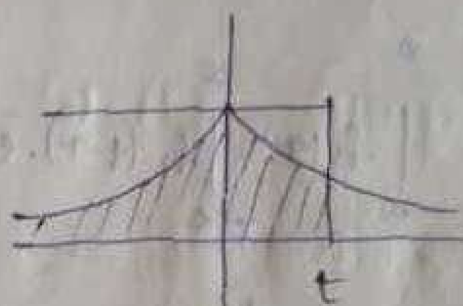
$$\int_{-\infty}^{t} 1\cdot e^{\tau}\cdot d\tau$$



$e^{(t+\infty)}$

Case-II : $t > 0$

$$\int_{-\infty}^{0} e^{\tau}\cdot d\tau + \int_{0}^{t} e^{-\tau} d\tau$$



$\cancel{e^{(0+\infty)}} + \cancel{e^{(t-0)}}$

$\cancel{e^{(0+\infty)}} - e^{-(t-0)}$

$\cancel{e^{(0+\infty)}} - \cancel{e^{t}}\cdot e^{0}$

$e^{0}-e^{-\infty}-(e^{-t}-e^{0})$

$\cancel{e^{\infty}} e^{-t}$

$\cancel{e^{\infty}}\ 1-e^{-t}+1 = 2-e^{-t}$

Convolution using fourier Transforms :-

$$f_1(t) \xleftrightarrow{FT} F_1(\omega)$$

$$f_2(t) \xleftrightarrow{FT} F_2(\omega)$$

$$f_1(t) * f_2(t) \xleftrightarrow{FT} F_1(\omega) \cdot F_2(\omega)$$

means, convolution in time domain is equal to multiplication in frequency domain

$$FT\{f_1(t) * f_2(t)\} = \int_{-\infty}^{\infty} [f_1(t) * f_2(t)] \cdot e^{-j\omega t} \, dt$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \{ f_1(\tau) \cdot f_2(t-\tau) \} d\tau \right] \cdot e^{-j\omega t} \, dt$$

$$= \int_{-\infty}^{\infty} f_1(\tau) \cdot \int_{-\infty}^{\infty} f_2(t-\tau) \cdot e^{-j\omega t} \, dt \cdot d\tau$$

From time shifing property

$$f(t-t_0) \longleftrightarrow e^{-j\omega t_0} F(\omega)$$

$$\int_{-\infty}^{\infty} f_1(\tau) \cdot e^{-j\omega\tau} F_2(\omega) \cdot d\tau$$

$$F_2(\omega) \cdot \int_{-\infty}^{\infty} f_1(\tau) \cdot e^{-j\omega\tau} \, d\tau \implies F_1(\omega) \cdot F_2(\omega)$$

Find the convolution between $e^{-t}u(t)$ and $e^{-2t}u(t)$ using fourier transforms?

$$f_1(t) * f_2(t) \longleftrightarrow F_1(\omega)\cdot F_2(\omega)$$

$$F_1(\omega) = \frac{1}{1+j\omega}$$

$$F_2(\omega) = \frac{1}{2+j\omega}$$

$$F_1(\omega)\cdot F_2(\omega) = \frac{1}{1+j\omega} \times \frac{1}{2+j\omega}$$

$$F^{-1}\{F_1(\omega)\cdot F_2(\omega)\} = \frac{A}{1+j\omega} + \frac{B}{2+j\omega}$$

$$f_1(t) * f_2(t) = \frac{1}{1+j\omega} + \frac{-1}{2+j\omega} \qquad A = 1,\ B = -1$$

$$= e^{-t}u(t) - e^{-2t}u(t)$$

$$= \cancel{e^{-t}u(t)} - (e^{-t} - e^{-2t})u(t)$$

find ~~Write~~ the convolution b/w $f(t)$ and $\delta(t)$

$$\cancel{f(t)}\ F_1(\omega) = F(\omega)$$

$$F_2(\omega) =$$

$$F_1(\omega) \cdot F_2(\omega) = F(\omega) \cdot 1$$

$$F^{-1}\{F_1(\omega) \cdot F_2(\omega)\} = f(t)$$

$$f_0(t) * \delta(t) = f(t)$$

3. $f(t)$ and $\delta(t-t_0)$

$$f(t) \xleftrightarrow{FT} F(\omega)$$

$$\delta(t-t_0) \longleftrightarrow e^{-j\omega t_0} \quad \delta(\omega) \rightarrow e^{-j\omega t_0}\ (1)$$

$$F_1(\omega) \cdot F_2(\omega) = F(\omega) \cdot e^{-j\omega t_0}$$

$$F^{-1}\{F_1(\omega) \cdot F_2(\omega)\} = F^{-1}\{F(\omega) \cdot e^{-j\omega t_0}\}$$

$$f_1(t) * \delta(t-t_0) = f(t-t_0)$$

4. $e^{-2t} u(t) * \delta(t-4)$

$$e^{-2t} u(t) \longleftrightarrow \frac{1}{2+j\omega}$$

$$\delta(t-4) \longleftrightarrow e^{-j4\omega}$$

$$F_1(\omega) \cdot F_2(\omega) = \frac{1}{2+j\omega} \cdot e^{-4j\omega}$$

$$F^{-1}\{F_1(\omega) \cdot F_1(\omega)\} = F^{-1}\left\{\frac{e^{-4j\omega}}{2+j\omega}\right\}$$

$e^{-2(t-4)} f(t-4) = e^{-2(t-4)} u(t-4)$

5. $f(t) \xleftrightarrow{FT} F(\omega)$ then find the FT of $\frac{d^2}{dt^2} f(2t-4) \longleftrightarrow ?$

$$f(t) \longleftrightarrow F(\omega)$$

$$\frac{d^2}{dt^2} f(2t-4) \longleftrightarrow$$

$$f(2t) \longleftrightarrow \frac{1}{2} F(\omega/2) \quad [\text{Time scaling}]$$

$$f(2t-4) \longleftrightarrow e^{-j\omega 4} \cdot \frac{1}{2} F(\omega/2) \quad [\text{Time shifting}]$$

$$\frac{d^2}{dt^2} f(2t-4) \longleftrightarrow (j\omega)^2 e^{-j\omega 4} \frac{1}{2} F(\omega/2)$$

[differentitation in time domain]

Multiplication property of FT (or) convolution in frequency domain :-

$$f_1(t) \longleftrightarrow F_1(\omega)$$

$$f_2(t) \longleftrightarrow F_2(\omega)$$

$$2\pi f_1(t) f_2(t) \longleftrightarrow F_1(\omega) * F_2(\omega)$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} F_1(\omega) \cdot F_2(u-\omega) \cdot du \right] \cdot e^{j\omega t} \cdot dt$$

$$= \int_{-\infty}^{\infty} F_1(u) \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} F_2(u-\omega) \cdot e^{j\omega t}\, d\omega \right] \cdot du$$

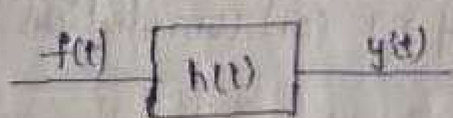
$$F(\omega - \omega_0) \longleftrightarrow e^{j\omega_0 t} \cdot f(t)$$

$$= f_2(t) \cdot \int_{-\infty}^{\infty} F_1(u) \cdot e^{jut}\, du$$

$$= f_2(t) \cdot 2\pi \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(u) \cdot e^{jut} \cdot du$$

$$= 2\pi\, f_1(t) \cdot f_2(t)$$

Transfor function of a system :-



$$y(t) = f(t) * h(t)$$

Apply fourier transform

$$Y(\omega) = F(\omega) \cdot H(\omega)$$

$$\boxed{H(\omega) = \frac{Y(\omega)}{F(\omega)}}$$

Where $H(\omega)$ is called the transfor function of system

$h(t) \xleftrightarrow{FT} H(\omega)$

↓ Impulse response

↓ Transfor f^n of system.

Filter characterstics of a system :-

we know that

$$y(t) = x(t) * h(t)$$

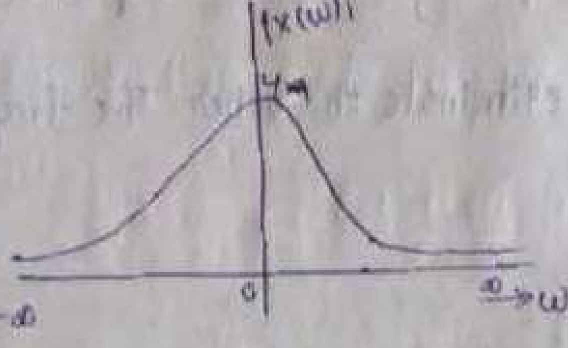
$$Y(\omega) = X(\omega) \cdot H(\omega) \quad \text{--- (I)}$$

considering magnitudes

$$|Y(\omega)| = |X(\omega)| \cdot |H(\omega)|$$

consider a signal $e^{-at} u(t)$ with fourier transform $\frac{1}{a+j\omega}$

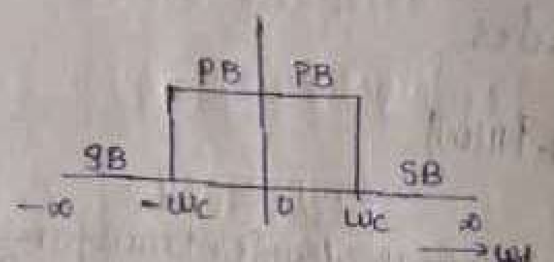
the magnitude response is as shown



From Equation(I) it is clear that selecting magnitude $|H(\omega)|$ can deside frequency components of $|X(\omega)|$ to exsist in $|Y(\omega)|$. It meanse in some sense $H(\omega)$ is performing filtering action. Based on the spectrum (frequency response) of $H(\omega)$, their are 4 types of filters. They are

1. low pass filter (LPF):

It allows all the frequency components less than the cut off region ω_c and attenuate/eliminate the other. The frequency response is as shown.

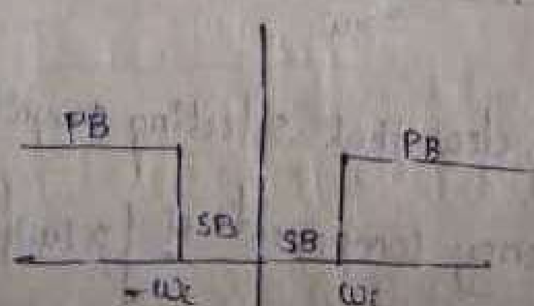


PB → pass band

SB → stationary band

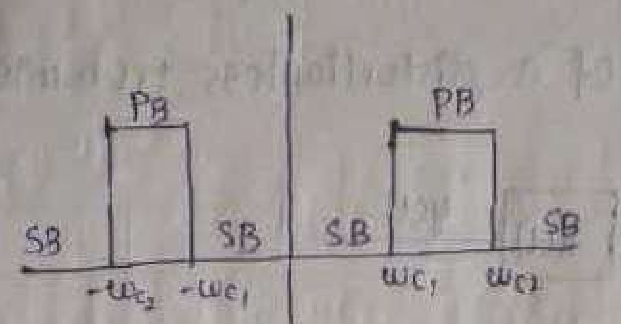
2. High pass filter:

It allows all the frequency components greater than the cut off ω_c and eliminate the other. The frequency response is as shown.



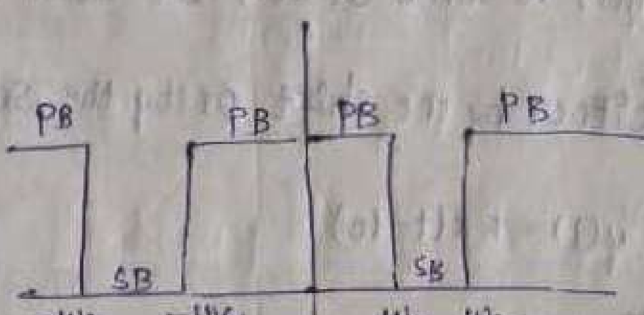
3. Band pass filter:

It allows all the frequency components between the cut off (ω_c) and eliminate other. The frequency response is as shown



4. Band Reject filter:

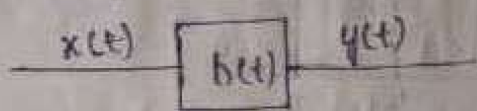
It rejects all the frequency components between the cut off and allows other. The frequency response is as shown



Distortionless transmission system :-

The system is said to be distortionless if the shape of the signal is preserved that means the shape of the output is same as the shape of the input

Transfor function of a distortionless transmission :-



$$y(t) = x(t) * h(t)$$

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

For a system to be called as distortionless as long as the shape of y(t) is same as x(t). It doesn't effect the shape either by the shift or by the scaling factor, therefore $y(t) = K x(t - t_0)$

$$Y(\omega) = K \cdot e^{-j\omega t_0} \cdot X(\omega)$$

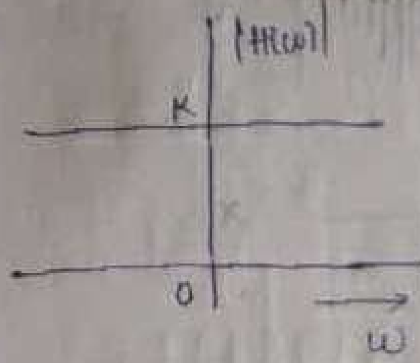
therefore the transfor function $H(\omega) = \frac{K \cdot e^{-j\omega t_0} \cancel{X(\omega)}}{\cancel{X(\omega)}}$

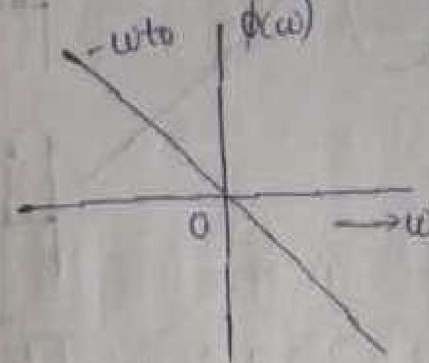
$$H(\omega) = K \cdot e^{-j\omega t_0} \quad \text{--- (1)}$$

From (i) the magnitude response of $H(\omega)$

$$|H(\omega)| = K$$

as the phase response $\phi(\omega) = -\omega t_0$ (from euler's identity)



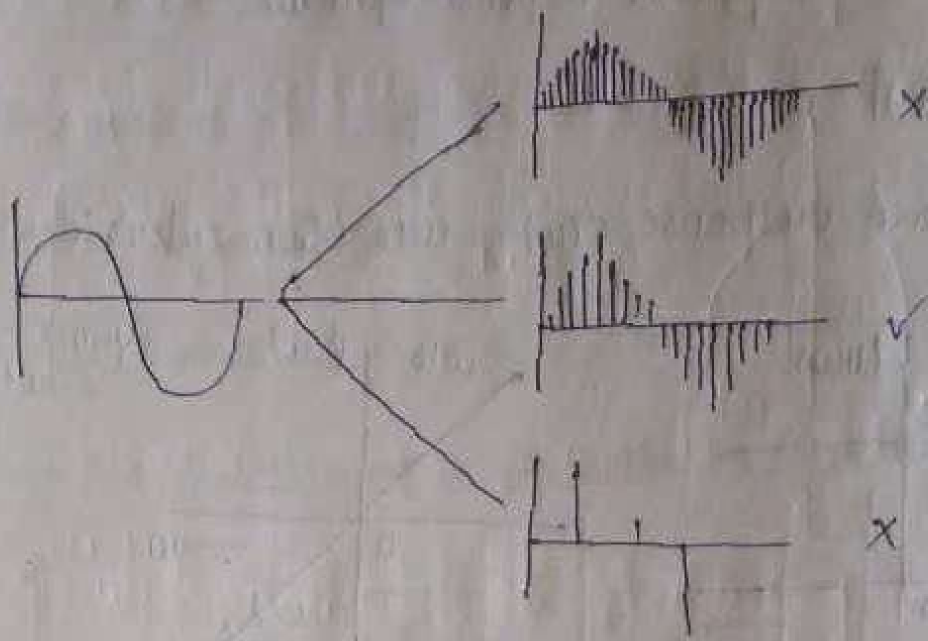


$y = mx$
$= -\omega t_0$

That means that magnitude response is constant while the phase response is linearity proportional to the shift

Sampling:

Sampling is a process, where we convert a continuous signal to discrete. This is possible by making use of sampling or equivalence property of an impulse signal, given by $f(t)\,\delta(t-t_0) = f(t_0)\cdot\delta(t-t_0)$



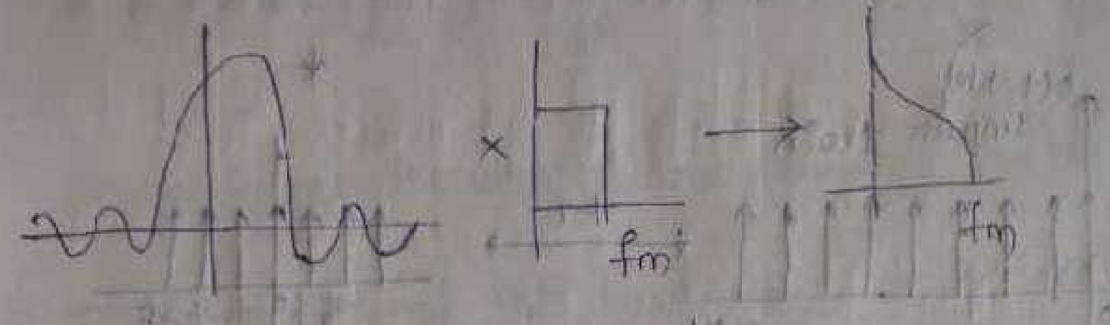
consider a signal $x(t)$ to be discretised. The problem here is in disciding the no. of samples to exist in the discrete signal. The sol^n for this problem is given by sampling theorem.

Sampling theorem:

The signal $x(t)$ band limited to f_m Hz can be represented interms of its Samples and can be reconstructed provided $f_s \geq 2f_m$ where f_s is the no. of samples and f_m is the band limited frequency of $f(t)$.

Band limiting:

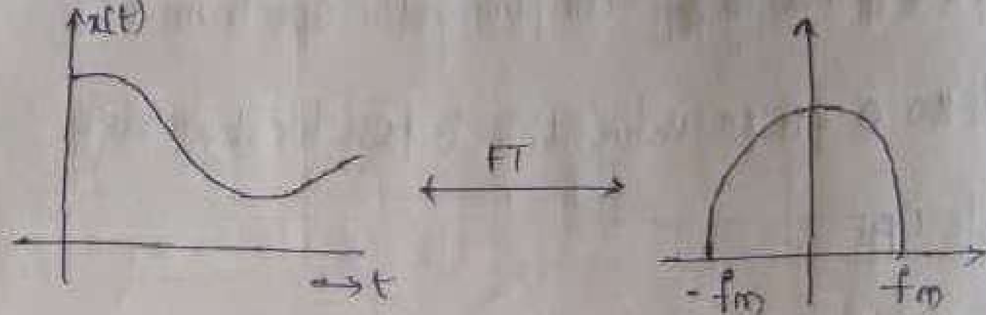
It is a process of confining the spectrum of a signal to a finite value this is possible by using a simple LPF

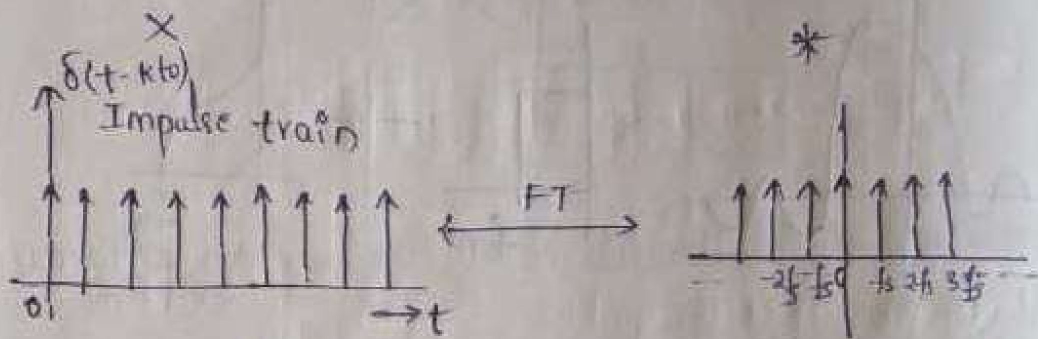


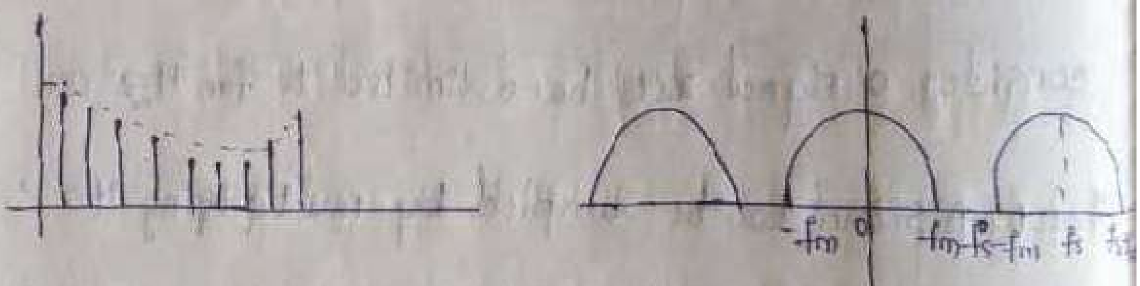
Proof:

consider a signal x(t) band limited to fm Hz as shown. A signal can be sampled by multiplying it with a periodic impulse train. The process of sampling and its impact in frequency domain is as depicted.

Time domain   frequency domain

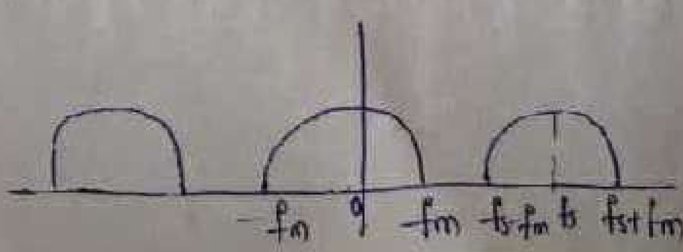




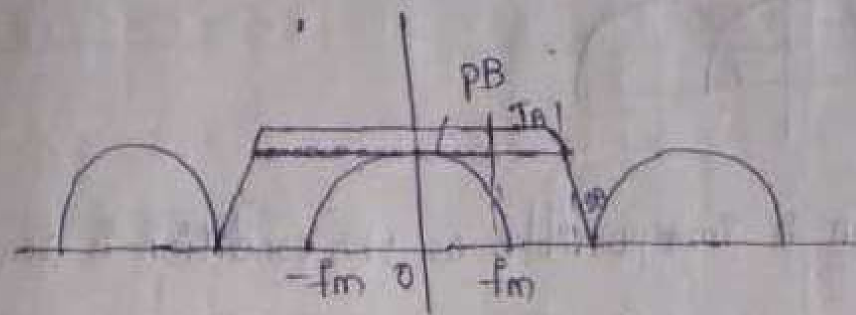


as a result of convolution in frequency domain, the spectrum turns periodic. Their arise 3 cases based on sampling frequency.

case-I : $f_s > 2f_m$

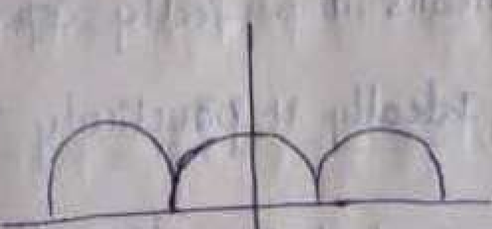


The signal can be reconstructed using a practical low pass filter.



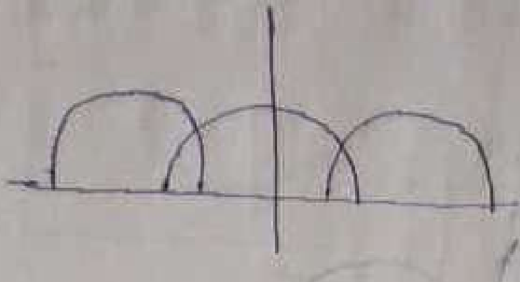
The gap between the wanted and unwanted spectra is called guard band. This band allows the transition band of a practical filter to settledown, before

case-II: $f_s = 2f_m$



The reconstruction is possible but one needs an ideal filter to accumplish this. $F_S = 2F_m$ is called as Nyqist rate of sampling.

Case-III :- $f_s < 2f_m$

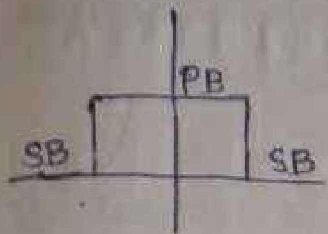


Reconstruction is impossible as this condition is results in spectral overlapping. This phenomenon is called -Aliasing.

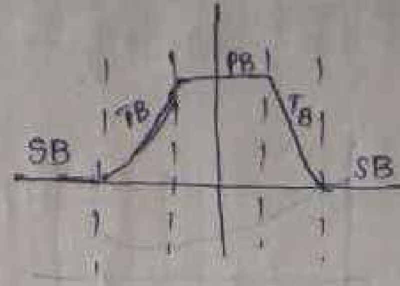
Ideal vs practical filters :-

All Ideal filters have very sharpe cut off near the cut-off frequency. It means it perfectly seperates stop band and pass band. Ideally it practicaly it is impossible to have such a perfect seperation between pass band and stop band. All practical filters take some time to change from pass band to stop band or SB to PB, that duration is called Transition band.

Ideal



Practical.



Types of Sampling :-

There are 3 types

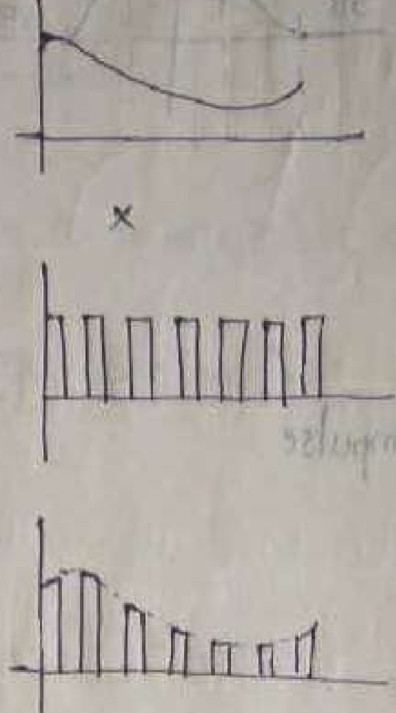
1. Ideal Sampling (or) Impulse
2. Natural sampling
3. Flat-Top sampling.

1. Ideal sampling :-

The type of sampling where the message is sampled by impulse is called ideal sampling. This is practically not possible.

2. Natural sampling :-

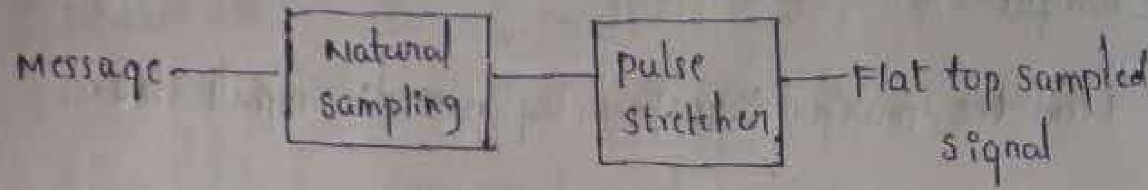
The type of sampling where the message signal is sampled by pulse is called as Natural sampling. The pulses selected here are with width as minimum as possible such that they closely resumble an impulse.



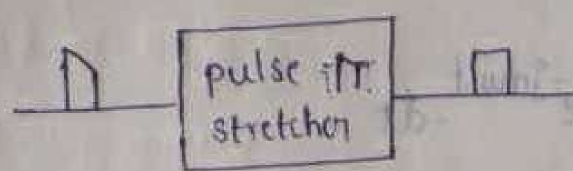
The problem with natural sampling is that the pulses take the shape of the message. This always creates a problem during reconstruction

3. Flat-Top sampling :-

unlike natural sampling, the shape of the pulse after sampling is made flat. This is possible by making use of a device called pulse strecher



The function of pulse stretcher is to randomly assign a value between the two limits of a pulse which is naturally sampled. The pulse is assigned that value with a constant width.



Fourier transform of periodic signals:-

Fourier transform is also aplicable for periodic signals. consider a signal which is periodic. It is represented in fourier series as

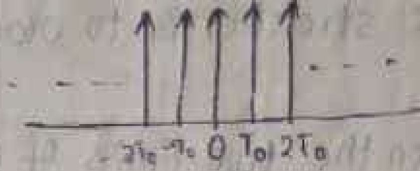
$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

applying fourier transform for the above we get

$$F(\omega) = 2\pi \sum_{n=-\infty}^{\infty} C_n \, \delta(\omega - n\omega_0)$$

Example:

Find the fourier transform of periodic impulse train.



Let a signal is represented as $x(t) = \delta(t - kT_0)$

Where $k = \pm Z$

$$C_n = \frac{1}{T_0}\int_{\langle T_0 \rangle} \delta(t - kT_0)\, e^{-jn\omega_0 t} \cdot dt$$

$$C_n = \frac{1}{T_0}\int_{\langle T_0 \rangle} \delta(t) \cdot e^{-jn\omega_0 t} \cdot dt$$

put $t = 0$ $\qquad \int f(t) \cdot \delta(t - t_0) \cdot dt = f(t_0)$

$$C_n = \frac{1}{T_0}\int_{\langle T_0 \rangle} \delta(t) \cdot e^{0} \cdot dt$$

$$\boxed{C_n = \frac{1}{T_0}}$$

$$F(\omega) = 2\pi \sum_{n=-\infty}^{\infty} C_n \, \delta(\omega - n\omega_0)$$

$$F(\omega) = 2\pi \sum_{n=-\infty}^{\infty} \frac{1}{T_0} \delta(\omega - n\omega_0)$$

$$= \frac{2\pi}{T_0} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

$$\boxed{F(\omega) = \omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)}$$

Laplace transform:-

In process of converting a signal from time domain to frequency domain using fourier transform, there were few signals which were not convergent and hence were not transformable. Laplace transform works in such cases where fourier fails. This is possible by including a term $e^{-\sigma t}$ in expression of Fourier transform. The expression becomes as shown

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} \cdot dt \quad \text{——(1)}$$

$$Lap\{f(t)\} = \int_{-\infty}^{\infty} \left(f(t) \cdot e^{-\sigma t}\right) \cdot e^{-j\omega t} \cdot dt \quad \text{——(2)}$$

it means finding laplace for $f(t)$ is nothing but to find Fourier for $f(t) \cdot e^{-\sigma t}$.

$$Lap\{f(t)\} = \int_{-\infty}^{\infty} f(t)$$

From (1) for the transform to exist $\int_{-\infty}^{\infty} f(t) \cdot dt < \infty$

——(3)

from (2) for the transform to exist

$$\int_{-\infty}^{\infty} f(t) \cdot e^{-\sigma t} \cdot dt < \infty \quad \text{--- (4)}$$

from (3) and (4), signals which are not convergent from (3) can be made convergent by appropriatly selecting the values of 'σ'

example

for u(t), the fourier transform to exist

$$\int_{-\infty}^{\infty} u(t) \cdot dt = \int_{0}^{\infty} 1 \cdot dt$$

$$= \infty \text{ (fourier fails)}$$

for the same signal, the laplace to exist

$$\int_{-\infty}^{\infty} u(t) \cdot e^{-\sigma t} \cdot dt = \int_{0}^{\infty} e^{-\sigma t} \cdot dt$$

$$= \frac{1}{\sigma}$$

which is a constant, Hence, laplace exist.

From (2)

$$\text{lap}\{f(t)\} = \int_{-\infty}^{\infty} f(t) \cdot e^{-\sigma t} \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{-\infty}^{\infty} f(t) \cdot e^{-(\sigma + j\omega)t} \, dt$$

$$= \int_{-\infty}^{\infty} f(t) \cdot e^{-st} \cdot dt \quad (s = \sigma + j\omega)$$

$\therefore \text{lap}\{f(t)\} = F(s)$

$$\boxed{F(s) = \int_{-\infty}^{\infty} f(t) \cdot e^{-st} \cdot dt}$$

From (4), it is clear that the convergence of $f(t)$ is completly depending on the value of 'σ'. Therefore the range of 'σ' for which the signal converges and transform exist, is called as Roc (Region of convergence

Laplace transform of some signals :-

1. $e^{-at} u(t)$

$$F(s) = \int_{-\infty}^{\infty} e^{-at} u(t) \cdot e^{-\sigma t} e^{-j\omega t} \cdot dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-\sigma t} \cdot e^{-j\omega t} \cdot dt$$

$$\uparrow \int_0^\infty e^{-(a+\sigma)\cdot t} \cdot dt < \infty$$

$+ (-)(+) = -$

$a+\sigma > 0 \Rightarrow \sigma > -a$

$Re\{s\} > -a$

$$F(s) = \int_0^\infty e^{-(a+\sigma+j\omega)t} \cdot dt$$

$$= \int_0^\infty e^{-(a+s)\cdot t} \cdot dt$$

$$F(s) = \frac{1}{s+a}, \quad \sigma > -a$$

2. $e^{at} u(t)$

$$F(s) = \int_{-\infty}^\infty e^{at} u(t) \cdot e^{-\sigma t} \cdot e^{-j\omega t} \cdot dt$$

$$= \int_0^\infty e^{at} e^{-\sigma t} \cdot e^{-j\omega t} \cdot dt$$

$$\int_0^\infty e^{(a-\sigma)t} \cdot dt < \infty$$

$$= \int_0^\infty e^{-(\sigma-a+j\omega)t} \cdot dt$$

$(+)(+)(-) = -$

$a-\sigma < 0$

$\sigma > a$

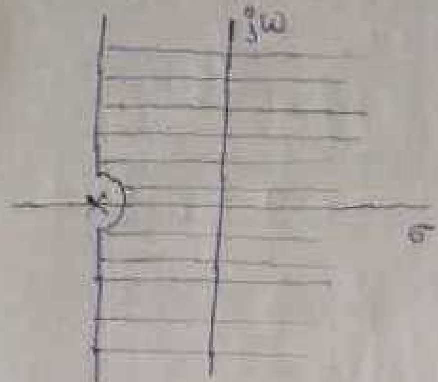
$$= \int_0^\infty e^{-(s-a)t} \cdot dt$$

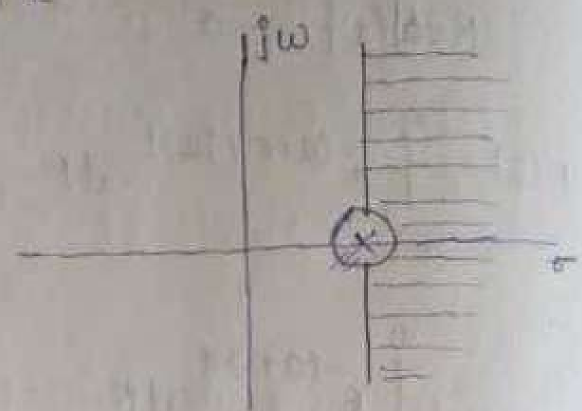
$$= \frac{1}{s-a} \quad Re\{s\} > a$$

Plotting ROC in S-plane :-

$e^{-at}u(t)$ $\qquad\qquad$ $e^{at}u(t)$

$\frac{1}{s+a}$, $Re\{s\} > -a$ $\qquad\qquad$ $\frac{1}{s-a}$, $Re\{s\} > a$





3. $e^{at}u(-t)$

$$F(s) = \int_{-\infty}^{\infty} e^{at}u(-t) \cdot e^{-\sigma t} \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{-\infty}^{0} e^{at} \cdot e^{-\sigma t} \cdot e^{-j\omega t} \cdot dt$$

$$\int_{-\infty}^{0} e^{(a-\sigma)t} \cdot dt < \infty$$

$(-)(+) = (-)$

$$= \int_{-\infty}^{0} e^{(a-\sigma-j\omega)t} \cdot dt$$

$a - \sigma > 0$

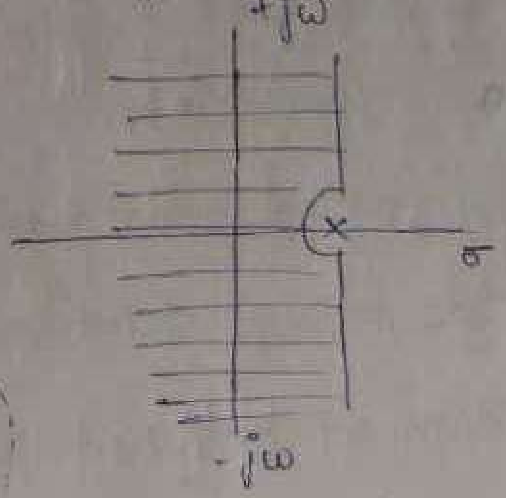
$\sigma < a$

$$= \int_{-\infty}^{0} e^{(a-s)t} \cdot dt \Rightarrow \left[\frac{e^{(a-s)t}}{a-s}\right]_{-\infty}^{0}$$

$$= \frac{1}{a-s}\,[1 - 0]$$

$$F(s) = \frac{1}{a-s} \quad , Re\{s\} < a$$

Plotting ROC in s-plane :-



4. $e^{-at} \cdot u(-t)$

$$F(s) = \int_{-\infty}^{\infty} e^{-at} u(-t) \cdot e^{-\sigma t} \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{-\infty}^{0} e^{-at} \cdot e^{-\sigma t} \cdot e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} e^{-(a+\sigma)t} \cdot dt$$

$$= \int_{-\infty}^{0} e^{(-a-\sigma-j\omega)t} \cdot dt$$

$(-)(-) = (+)$

$a + \sigma < 0$

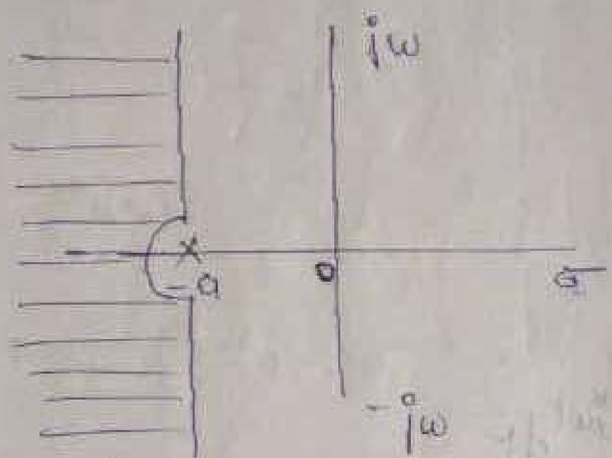
$\sigma < -a$

$$= \int_{-\infty}^{0} e^{(-s-a)t} \cdot dt$$

$$= \left[\frac{e^{(-s-a)t}}{-s-a} \right]_{-\infty}^{0}$$

$$= \frac{-1}{s+a} \;,\; Re\{s\} < -a$$

Plotting ROC:-



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$$e^{-at}u(t) \longleftrightarrow \frac{1}{s+a} \;,\; Re\{s\} > -a$$

$$e^{at}u(t) \longleftrightarrow \frac{1}{s-a} \;,\; Re\{s\} > a$$

$$-e^{at}u(-t) \longleftrightarrow \frac{1}{s-a} \;,\; Re\{s\} < a$$

$$-e^{-at}u(-t) \longleftrightarrow \frac{1}{s+a} \;,\; Re\{s\} < -a$$

poles and zeros of a system

consider a transfer function $H(s) = \frac{Y(s)}{X(s)}$ as shown

$$H(s) = \frac{K(s+z_1)(s+z_2)\cdots}{(s+p_1)(s+p_2)\cdots}$$

The denominator of transfer function is called as characteristic equation and its roots are called as poles of the system. The roots of the numerator of $H(s)$ are called as zeros of the system.

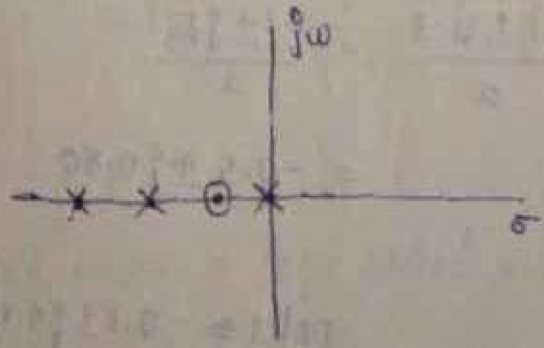
Example:-

$$H(s) = \frac{s+1}{s(s+2)(s+3)}$$

poles = $0, -2, -3 \rightarrow \times$

zeros = -1 — o

pole-zero plot :-

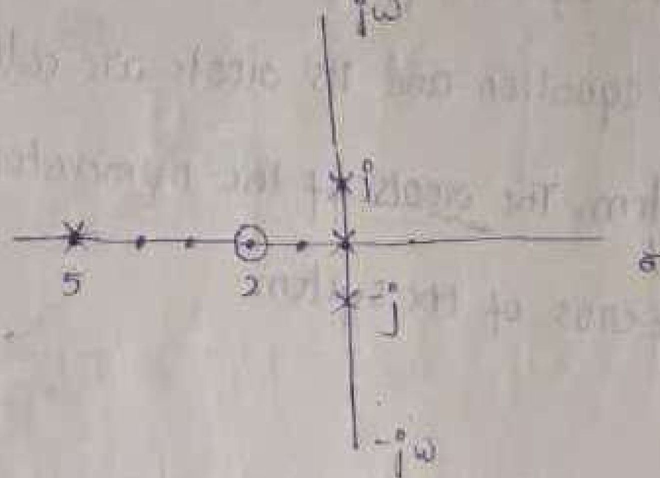


2. $H(s) = \dfrac{s+2}{s(s^2+1)(s+5)}$

Poles $= 0, \pm j, -5$

zeros $= -2$

pole-zero plot:-



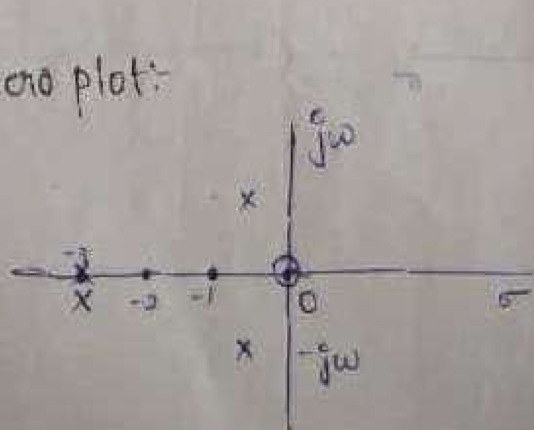
3. $H(s) = \dfrac{s}{(s^2+s+1)(s+3)^2}$

$s^2+s+1 \Rightarrow b=1, a=1, c=1$

$\dfrac{-1 \pm \sqrt{1-4(1)}}{2} = \dfrac{-1 \pm \sqrt{-3}}{2} = \dfrac{-1 \pm j\sqrt{3}}{2}$

$= -0.5 \pm j0.86$

pole-zero plot:-



poles $= -0.5 \pm j0.86, 3, 3$

zeros $= 0$

Properties of Roc :-

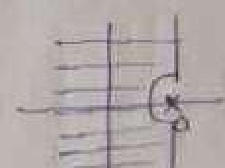
1. Roc is a stright line parallel to $j\omega$-axis

2. Roc should not include any pole.

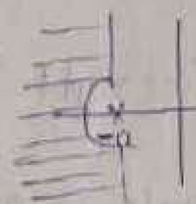
3. Roc of a left side signal is also left sided with respect to the pole.

ex:

$-e^{at}u(-t) = \frac{1}{s-a}$ $Re\{s\} < a$



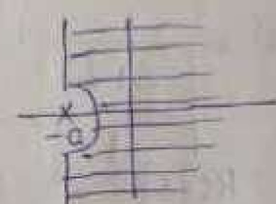
$-e^{-at}u(-t) = \frac{1}{s+a}$ $Re\{s\} < -a$



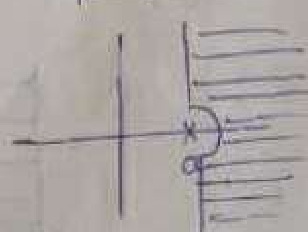
4. Roc of a right side signal is also right sided with respect to the pole

example:-

$e^{at}u(+t) = \frac{1}{s+a}$ $Re\{s\} > -a$



$e^{-at}u(t) - \frac{1}{s-a}$ $Re\{s\} > a$

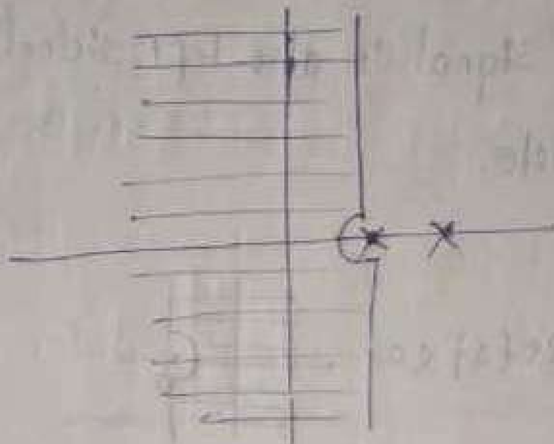


5. For multiple left sided signals, the Roc is w.r.t the pole of smallest value

Ex: $f(t) = -e^{2t}u(-t) - e^{3t}u(-t)$

$$F(s) = \frac{1}{s-2} + \frac{1}{s-3}$$

$Re\{s\} < 2 \quad Re\{s\} < 3$

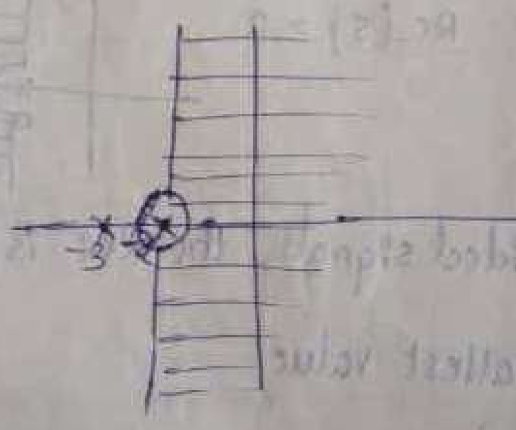


6. For multiple right-sided signals, the ROC is w.r.t to the pole of greatest value

Ex: $e^{-2t}u(t) + e^{-3t}u(t)$

$$F(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$Re\{s\} > -2 \quad Re\{s\} > 3$



→ For combination of left and right sided signals the common area is ROC

Note:-

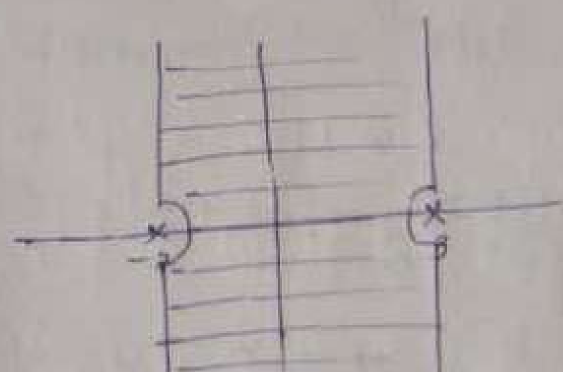
The transform doesn't exist if their is no common area possible.

Ex:

1. $e^{-2t}u(t) - e^{3t}u(-t)$

$$\frac{1}{s+2} + \frac{1}{s-3}$$

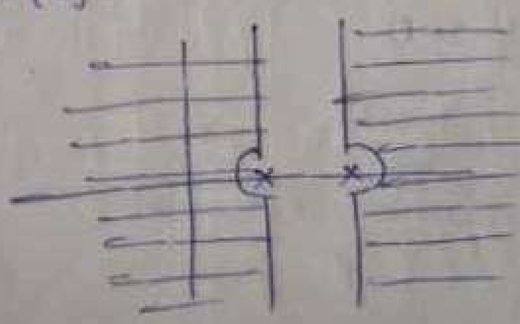
$Re\{s\} > -2 \quad Re\{s\} < 3$



2. $-e^{-2t}u(-t) + e^{3t}u(t)$

$$\frac{1}{s-2} + \frac{1}{s-3}$$

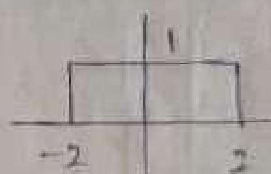
$Re\{s\} < 2 \quad Re\{s\} > 3$



Since, there is no common area, transform does not exist.

8. The ROC of a finite duration signal is entire s-plane except $s=0$ (or) $s=\infty$

Example:



$$\int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$\int_{-2}^{2} e^{-st}\, dt$$

$$\left[\frac{e^{-st}}{-s}\right]_{-2}^{2}$$

$$\frac{\cancel{e^{-2s} + e^{2s}}}{-\cancel{s}} \qquad \frac{e^{-2s} - e^{2s}}{-s}$$

Entire 's' plane except $s=0$

Properties of laplace transform

(i) Linearity property: The linearity property states that the L.T of a weighted sum of two signals is equal to of weighted sum of individual L.T's

$$x_1(t) \xleftrightarrow{LT} X_1(s) \qquad x_2(t) \xleftrightarrow{LT} X_2(s)$$

$$ax_1(t) + bx_2(t) \xleftrightarrow{LT} aX_1(s) + bX_2(s)$$

Proof:

$$L\{ax_1(t)+bx_2(t)\} = \int_{-\infty}^{\infty} [ax_1(t)+bx_2(t)].e^{-st}.dt$$

$$= a\int_{-\infty}^{\infty} x_1(t).e^{-st}.dt + b\int_{-\infty}^{\infty} x_2(t).e^{-st}.dt$$

$$L\{ax_1(t)+bx_2(t)\} = aX_1(s) + bX_2(s)$$

$$ax_1(t) + bx_2(t) \xleftrightarrow{LT} aX_1(s) + bX_2(s)$$

2) Time shift property: Time shifts property states that if $x(t) \xleftrightarrow{LT} X(s)$ then $x(t-t_0) \xleftrightarrow{LT} e^{-st_0} X(s)$

Proof: $$L[x(t-t_0)] = \int_{-\infty}^{\infty} x(t-t_0).e^{-st}.dt$$

let $t - t_0 = p \Rightarrow t = p + t_0$
$dt = dp$

$$L[x(t-t_0)] = \int_{-\infty}^{\infty} x(p) \cdot e^{-s(p+t_0)} \cdot dp$$

$$= \int_{-\infty}^{\infty} x(p) \cdot e^{-sp} \cdot e^{-st_0} dp$$

$$= e^{-st_0} \int_{-\infty}^{\infty} x(p) \cdot e^{-sp} \cdot dp$$

$$L[x(t-t_0)] = e^{-st_0} \cdot X(s) \quad \text{||ly} \quad L[x(t+t_0)] = e^{st_0} \cdot X(s)$$

③ time ~~scaling~~ reversal property :-

time ~~scaling~~ reversal property states that : $x(t) \xleftrightarrow{LT} X(s)$

then $x(-t) \xleftrightarrow{LT} X(-s)$

Proof :

$$L[x(-t)] = \int_{-\infty}^{\infty} x(-t) \cdot e^{-st} \cdot dt$$

put $-t = p$; if $t = -\infty, p = \infty$
$dt = -dp$; $t = \infty, p = -\infty$

$$= \int_{p=\infty}^{-\infty} x(p) \cdot e^{-s(-p)} (-dp)$$

$$L[x(-t)] = \int_{-\infty}^{\infty} x(p) \cdot e^{sp} \cdot dp$$

$$L[x(-t)] = X(-s)$$

$$x(-t) \xleftrightarrow{LT} X(s)$$

4. Time scaling property:

The time scaling property states that if $x(t) \xleftrightarrow{LT} X(s)$

then $x(at) \longleftrightarrow \frac{1}{|a|} X(s/a)$

Proof: $L[x(at)] = \int_{-\infty}^{\infty} x(at) \cdot e^{-st} \cdot dt$

put $at = p$

$a \cdot dt = dp$

$dt = dp/a$

$$L[x(at)] = \int_{-\infty}^{\infty} x(p) \cdot e^{-s(p/a)} \cdot dp/a$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} x(p) \cdot e^{-s(p/a)} \cdot dp$$

$$L[x(at)] = \frac{1}{|a|} X(s/a)$$

$$\therefore x(at) \xleftrightarrow{LT} \frac{1}{|a|} X(s/a)$$

5. Transform of Derivaties property:

Transform of derivaties property states that if

$x(t) \xleftrightarrow{LT} X(s)$ then $\frac{d}{dt}x(t) \xleftrightarrow{LT} sX(s) - x(0^-)$

Proof:

$$L\left[\frac{d}{dt}x(t)\right] = \int_{+0}^{\infty}\underbrace{\left[\frac{d}{dt}x(t)\right]}_{v} \cdot \underbrace{e^{-st}}_{u} dt$$

ILATE, u/v

$$= e^{-st}\int_{+0}^{\infty}\frac{d}{dt}x(t)\cdot dt - \int_{+0}^{\infty}\frac{d}{dt}\left[\frac{d}{dt}(x(t))\right]\int e^{-st}dt\cdot dt$$

$$= \left[e^{-st}x(t)\right]_{0}^{\infty} + \int_{-0}^{\infty} s e^{-st}x(t)dt$$

$$= \left[e^{-\infty}x(\infty) - e^{0}x(0^-)\right] + s\int_{0}^{\infty}x(t)\cdot e^{-st}\cdot dt$$

$$= -x(0^-) + sX(s)$$

$$\frac{d}{dt}x(t) \xleftrightarrow{LT} sX(s) - x(0^-)$$

Similarly $\frac{d^2x(t)}{dt^2} \xleftrightarrow{LT} s^2X(s) - sx(0^-) - s^0\frac{d}{dt}x(0^-)$

$$L\left[\frac{d^nx(t)}{dt^n}\right] = s^nX(s) - s^{n-1}x(0^-) - s^{n-2}x(0^-)\dots - \frac{d^{n-1}x(0^-)}{dt^{n-1}}$$

6. Transforms of Integrals property (or) Integration in time domain

$x(t) \xleftrightarrow{LT} X(s)$

$\int x(t)dt \xleftrightarrow{LT} \frac{X(s)}{s}$

$$L\left[\int x(t).dt\right] = \int_{-\infty}^{\infty} \underbrace{\int x(t).dt}_{u} \cdot \underbrace{e^{-st}}_{v} .dt$$

$$= \left[\int x(t).dt \cdot \frac{e^{-st}}{-s}\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} x(t) \frac{e^{-st}}{-s} .dt$$

$$= 0 + \frac{1}{s} X(s)$$

$$L\left[\int x(t).dt\right] = \frac{X(s)}{s}$$

7. Differentiation in S-domain property

$x(t) \xleftrightarrow{LT} X(s)$

$-t x(t) \longleftrightarrow$

$$L\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} \cdot dt$$

differentitating both sides w.r.t to s

$$\frac{d}{ds} X(s) = \frac{d}{ds}\left[\int_{-\infty}^{\infty} x(t)\, e^{-st} dt\right]$$

$$= \int_{-\infty}^{\infty} x(t) \cdot \frac{d}{ds} e^{-st} \cdot dt$$

$$= \int_{-\infty}^{\infty} x(t) \cdot (-t) e^{-st} \cdot dt$$

$$= -\int_{-\infty}^{\infty} x(t) \cdot t \cdot e^{-st} \cdot dt$$

$$\therefore \quad L\{x(t) \cdot t\} = -\frac{d}{ds} X(s)$$

8. Integration in s-domain

$$x(t) \xleftrightarrow{LT} X(s)$$

$$\frac{x(t)}{t} \longleftrightarrow$$

Convolution in time domain:

$$x_1(t) \xleftrightarrow{L} X_1(s) \quad \text{ROC}: R_1$$

$$x_2(t) \xleftrightarrow{L} X_2(s) \quad \text{ROC}: R_2$$

$$x_1(t) * x_2(t) \xleftrightarrow{L} X_1(s) \cdot X_2(s) \quad \text{ROC}: R_1 \cap R_2$$

We have

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) . x_2(t-\tau) . d\tau$$

$$L\left[x_1(t) * x_2(t)\right] = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} x_1(\tau) . x_2(t-\tau) . d\tau \right\} . e^{-st} dt$$

$$= \int_{-\infty}^{\infty} x_1(\tau) . d\tau . \int_{-\infty}^{\infty} x_2(t-\tau) . e^{-st} . d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau) . d\tau . e^{-s\tau} . X_2(s)$$

$$= X_2(s) . \int_{-\infty}^{\infty} x_1(\tau) . e^{-s\tau} . d\tau$$

$$L\left[x_1(t) * x_2(t)\right] = X_1(s) . X_2(s)$$

Q:- find laplace transform of the following:

1. cosat u(t)

$$= \int_{-\infty}^{\infty} \cos at\, u(t) \cdot e^{-\sigma t} e^{-j\omega t} \cdot dt$$

$$= \int_{0}^{\infty} e^{-\sigma t} \cdot \cos at\, dt < \infty$$

$$\sigma > 0$$

$$\cos at = \frac{e^{jat} + e^{-jat}}{2}$$

$$L(\cos at) = \frac{1}{2}\left[L\left\{e^{+jat}u(t) + e^{-jat}u(t)\right\}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-ja} + \frac{1}{s+ja}\right]$$

$$= \frac{1}{2}\left[\frac{2s}{s^2+a^2}\right]$$

$$L(\cos at) = \frac{s}{s^2+a^2}, \quad \sigma > 0, \ Re\{s\} > 0$$

Note: For all sinusoidal waveforms, the real part is greater than zero

2. $\sin at\, u(t)$

$$\int_{-\infty}^{\infty} \sin at\, u(t) . e^{-\sigma t} . e^{-j\omega t} dt$$

$$\int_{0}^{\infty} \sin at . e^{-\sigma t} . dt > \infty$$

$\sigma > 0$

$$\sin at = \frac{e^{jat} - e^{-jat}}{2j}$$

$$L(\sin at) = \frac{1}{2j}\left[L\left\{ e^{jat}u(t) - e^{-jat}u(t) \right\} \right]$$

$$= \frac{1}{2j}\left[\frac{1}{s-ja} - \frac{1}{s+ja} \right]$$

$$= \frac{1}{\cancel{2j}}\left[\frac{\cancel{2j}a}{s^2+a^2} \right]$$

$$L(\sin at) = \frac{a}{s^2+a^2}, \quad \sigma > 0, \ Re\{s\} > 0$$

3. $\cosh at \, u(t)$

$$\int_{-\infty}^{\infty} \cosh at \, u(t) \cdot e^{-\sigma t} \cdot e^{-j\omega t} \, dt$$

$$\int_{0}^{\infty} e^{-\sigma t} \cdot \cosh at \cdot dt$$

$\sigma > 0$

$$\cosh at = \frac{e^{at} + e^{-at}}{2}$$

$$L(\cosh at) = \frac{1}{2}\left[L\left\{ e^{at}u(t) + e^{-at}u(t) \right\} \right]$$

$$= \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a} \right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}s}{s^2-a^2} \right]$$

$\cosh at\, u(t) \longleftrightarrow \frac{s}{s^2-a^2} \quad Re\{s\} > 0$

4. $\sinh at\, u(t)$

$$= \int_{-\infty}^{\infty} \sinh at\, u(t)\, e^{-\sigma t} \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{0}^{\infty} e^{-\sigma t} \sinh at\, dt < \infty$$

$$\sinh at = \frac{e^{at} - e^{-at}}{2}$$

$$L[\sinh at] = \frac{1}{2} \left[L\{ e^{at} u(t) - e^{-at} u(t) \} \right]$$

$$= \frac{1}{2} \left[\frac{1}{s-a} - \frac{1}{s+a} \right]$$

$$= \frac{1}{\cancel{2}} \left[\frac{\cancel{2}a}{s^2-a^2} \right]$$

$$= \frac{a}{s^2-a^2} \; ; \; Re\{s\} > 0$$

5. $e^{-at}\cos bt\, u(t)$

$$= \int_{-\infty}^{\infty} e^{-at}\cos bt\, u(t) \cdot e^{-\sigma t} \cdot e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(\sigma+a)t} \cdot \cos bt \cdot dt < \infty$$

$\sigma + a > 0$ $\qquad \cos bt = \dfrac{e^{jbt} + e^{-jbt}}{2}$

$\sigma > -a$

$$= \int_{0}^{\infty} e^{\sigma} \quad L[e^{-at}\cos bt] = \frac{1}{2}\left[L\{e^{jbt-at}u(t) + e^{-jbt-at}u(t)\}\right]$$

$$= \frac{1}{2}\left[L\{e^{-(a-jb)t}u(t) + e^{-(jb+a)t}u(t)\}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s+a-jb} + \frac{1}{s+a+jb}\right]$$

$$= \frac{1}{2}\left[\frac{s+a+jb+s+a-jb}{(s+a)^2+b^2}\right]$$

$$= \frac{s+a}{(s+a)^2+b^2}$$

$\cos at\, u(t) \longleftrightarrow \frac{s}{s^2+a^2}$, $Re\{s\} > 0$

$\sin at\, u(t) \longleftrightarrow \frac{a}{s^2+a^2}$, $Re\{s\} > 0$

$\cosh at\, u(t) \longleftrightarrow \frac{s}{s^2-a^2}$, $Re\{s\} > 0$

$\sinh at\, u(t) \longleftrightarrow \frac{a}{s^2-a^2}$, $Re\{s\} > 0$

$e^{-at}\cos bt\, u(t) \longleftrightarrow \frac{s+a}{(s+a)^2+b^2}$, $Re\{s\} > -a$

$e^{-at}\sin bt\, u(t) \longleftrightarrow \frac{b}{(s+a)^2+b^2}$, $Re\{s\} > -a$

$e^{-at}\cosh bt\, u(t) \longleftrightarrow \frac{s+a}{(s+a)^2-b^2}$, $Re\{s\} > -a$

$e^{-at}\sinh bt\, u(t) \longleftrightarrow \frac{b}{(s+a)^2-b^2}$, $Re\{s\} > -a$

Find the laplace transform of u(t)

$$LT\{e^{-at}u(t)\} = \frac{1}{s+a}$$

$$\lim_{a\to 0} LT\{u(t)\} = 1/s$$

Find the laplace transform of t

$$t = \int u(t)$$

$$t = \frac{1}{s} \times \frac{1}{s}$$

$$t = 1/s^2$$

Inverse laplace transform:

$$X(s) = \frac{3s+7}{(s-3)(s+1)}$$

$$\frac{3s+7}{(s-3)(s+1)} = \frac{A}{(s-3)} + \frac{B}{(s+1)}$$

$$3s+7 = A(s+1) + B(s-3)$$

$$3(-1)+7 = A(0) + B(-1-3)$$

$$4 = B(-4)$$

$$B = -1$$

$$3s+7 = As + A + Bs - 3B$$

compare s coeffi.

$$3 = A + B$$

$$3 - B = A$$

$$7 = A - 3B$$

$$7 = 3 - B - 3B$$

$$4 = -4B$$

$$B = -1$$

put $s-3=0 \Rightarrow s=3$

$$3(3)+7 = A(3+1) + B(3-3)$$

$$16 = (4)A$$

$$A = 4$$

$$= \frac{4}{s-3} + \frac{-1}{s+1}$$

$$X(s) = \frac{-1}{s+1} + \frac{4}{s-3}$$

1. $Re\{s\} < -1$ (left)

$e^{-t}u(-t) - 4e^{3t}u(-t)$

2. $Re\{s\} > 3$

3. $-1 < Re\{s\} < 3$

1) $e^{-t}u(-t) - e^{-3t}u(-t)$

2) $e^{-t}u(t) + e^{3t}u(t)$

3) $e^{-3t}u(-t) - e^{-t}u(t)$

$$X(s) = \frac{s^2+2s-2}{s(s+2)(s-3)}$$

$$\frac{s^2+2s-2}{s(s+2)(s-3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s-3}$$

$$= A(s+2)(s-3) + B(s)(s-3) + C(s)(s+2)$$

$$= A(s^2-3s+2s-6) + B(s^2-3s) + C(s^2+2s)$$

$$= A(s^2-s-6)+B(s^2-3s)+C(s^2+2s)$$

$$s^2+2s-2 = As^2 - As - 6A + s^2B - 3Bs + Cs^2 + 2sC$$

$1 = A + B + C$

$1 = \frac{1}{3} + B + C$

$B + C = 2/3$ —(1)

$2 = -A - 3B + 2C$

$2 = -\frac{1}{3} - 3B + 2C$

$7/3 = -3B + 2C$ —(2)

$-2 = -6A$

$A = 2/6$

$A = 1/3$

$3B + 3C = 2$

$-3B + 2C = 7/3$

$5C = 2 + 7/3$

$5C = 13/3$

$C = 13/15$

$1 = A + B + C$

$1 = \frac{1}{3} + B + \frac{13}{15}$

$1 = \frac{18}{15} + B$

$B = -1/5$

$$X(s) = \frac{\frac{1}{3}}{s} + \frac{-\frac{1}{5}}{s+2} + \frac{\frac{13}{15}}{s-3}$$

$$= \frac{1}{3}u(t) - \frac{1}{5}e^{-2t}u(t) + \frac{13}{15}\cdot e^{3t}u(t) \quad (1)$$

$$= -\frac{1}{3}u(-t) + \frac{1}{5}e^{-2t}u(-t) - \frac{13}{15}e^{3t}u(-t) \quad (2)$$

(1) $Re\{s\} > 3$

(2) $Re\{s\} < -2$

$$X(s) = \frac{1}{s^2+3s+2}$$

$$\frac{1}{s^2+3s+2}$$

$$s^2+3s+2$$
$$= s^2+2s+s+2$$
$$= s(s+2)+1(s+2)$$
$$= (s+1)(s+2)$$
$$= \cancel{A(s+1)+B(s)}$$

$$\frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$1 = A(s+2) + B(s+1)$$

comparing co-efficents of s

$$0 = A+B$$
$$-A = -B$$
$$-A = -(-1)$$
$$\boxed{-A = 1}$$

comparing the constants

$$1 = 2A+B$$
$$1 = 2(-B)+B$$
$$\boxed{B = -1}$$

1) $x(t) = -e^{-t}u(-t) + e^{-2t}u(-t)$ $Re\{s\} < -2$

2) $x(t) = e^{-t}u(t) - e^{-2t}u(t)$ $Re\{s\} > -1$

3) $x(t) = e^{-2t}u(t) - e^{-t}u(-t)$ $-2 < Re\{s\} < -1$

① $x(s) = \dfrac{2s+1}{(s+2)(s-4)}$

$$\frac{2s+1}{(s+2)(s-4)} = \frac{A}{s+2} + \frac{B}{s-4}$$

$$2s+1 = A(s-4) + B(s+2)$$

$$2s+1 = As - 4A + Bs + 2B$$

comparing coeff of s

$$2 = A + B \quad \text{---} ①$$

comparing co-efficients of constants

$$1 = -4A + 2B \quad \text{---} ②$$

sloving (1) & (2)

$$4 = 2A + \cancel{2B}$$

$$\underline{\overset{(-)}{1} = \overset{(-)}{-4A} + \overset{(-)}{\cancel{2B}}}$$

$$3 = 6A$$

$\boxed{A = 1/2}$ (sub in (1))

$$A + B = 2$$

$$\frac{1}{2} + B = 2$$

$$B = 2 - \frac{1}{2}$$

$$\boxed{B = 3/2}$$

$$X(s) = \frac{1/2}{s+2} + \frac{3/2}{s-4}$$

$-\frac{1}{2} \cdot e^{-2t} u(t) + \frac{3}{2} e^{4t} \cdot u(t)$ — ① $Re\{s\} > 4$

$-1/2 \cdot e^{-2t} u(-t) - \frac{3}{2} e^{4t} u(-t)$ — ② $Re\{s\} < -2$

$1/2 e^{-2t} u(t) - \frac{3}{2} \cdot e^{4t} u(t)$ — ③ $-2 < Re\{s\} <$

② $X(s) = \frac{4}{s(s-1)(s+3)}$

$$\frac{4}{s(s-1)(s+3)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s+3}$$

$4 = A(s-1)(s+3) + Bs(s+3) + C s(s-1)$

$4 = A(s^2+3s-s-3) + B(s^2+3s) + C(s^2-s)$

$4 = As^2 + 9As - 3A + Bs^2 + 3Bs + Cs^2 - Cs$

s^2 coeff	s coeff	constants
$0 = A+B+C$ — ①	$0 = +9A+3B-C$ — ②	$4 = -3A - C$
$A+B+3A+4=0$	$+9A+3B-3A-4=0$	$-4 = 3A + C$
$4A+B=-4$ — ③	$+6A+3B=4$ — ④	$C = 3A+4$ — sub in ① & ②
slove (3) & (4)		

$12A + 3B = -12$

$6A + 3B = 4$

$A = -4/3$, $B = \frac{-4}{3}$ $C = 8/3$

$$X(s) = \frac{s}{(s+1)(s+3)}$$

$$\frac{s}{(s+1)(s+3)} = \frac{A}{(s+1)} + \frac{B}{(s+3)}$$

$$s = A(s+3) + B(s+1)$$

$1 = A + B$ —— (1) | $0 = 3A + B$ —— (2)

slove (1) & (2)

$$3A + B = 0$$
$$A + B = 1$$
$$(-) \quad (-) \quad (-)$$
$$2A = -1$$
$$A = -1/2 \quad (\text{sub in } (1)$$

$1 = A + B$

$1 = \frac{-1}{2} + B$

$B = 3/2$

$$X(s) = \frac{-1/2}{s+1} + \frac{3/2}{s+3}$$

$-1, -3$

$Re\{s\} > -1$

$Re\{s\} < -3$

$-3 < Re\{s\} < -1$

1. $\bar{e}^{1/2} \cdot e^{t} u(t) + 3/2 \cdot e^{-t} u(t)$

2. $\frac{-1}{2} \cdot \bar{e}^{t} u(-t) + \frac{3}{2} \bar{e}^{3t} u(-t)$

3. $-\frac{1}{2} e^{-t} u(t) - \bar{e}^{3t} u(-t)$

Find the inverse transform of $\frac{e^{-2s}}{s^2+3s+2}$

$$\frac{e^{-2s}}{s^2+3s+2} = \frac{1}{s^2+3s+2}$$

$$F(s) = \frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)}$$

$$= \frac{1}{(s+1)} + \frac{-1}{s+2}$$

$-1, -2$

1. $f(t) = -e^{-t}u(-t) + e^{-2t}u(-t)$ $\quad Re\{s\} < -2$

2. $f(t) = e^{-t}u(t) - e^{-2t}u(+t)$ $\quad Re\{s\} > -1$

3. $f(t) = e^{-2t}u(-t) + e^{-t}u(t)$ $\quad -2 < Re\{s\} < -1$

$\therefore$ F(s) multiplyied by e^{-2s}, f(t) will be shift by 2 units (from time shifting property)

$\therefore$ replace t by t-2 in (1), (2) and (3)

① $F(s) = -e^{-(t-2)}u(-(t-2)) + e^{-2(t-2)}u(-(t-2))$

② $F(s) = e^{-(t-2)}u(t-2) - e^{-2(t-2)}u(t-2)$

③ $F(s) = e^{-2(t-2)}u(-(t-2)) + e^{-(t-2)}u(t-2)$

2. $X(s) = \dfrac{s^2+2s+1}{(s+2)(s^2+4)}$

$$\frac{s^2+2s+1}{(s+2)(s^2+4)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+4}$$

$$s^2+2s+1 = A(s^2+4) + (Bs+C)(s+2)$$

put $s=-2$

$$4-4+1 = A(4+4)+0$$

$$1 = 8A$$

$$A = 1/8$$

$$s^2+2s+1 = As^2+4A+Bs^2+2Bs+Cs+2C$$

coefficent of s^2	coefficents of s	constants
$1 = A+B$	$2 = 2B+C$	$1 = 4A+2C$
		$1 = 4(1/8)+2C$
		$1-\frac{1}{2} = 2C$
		$\frac{1}{2} = 2C$
		$\boxed{C = 1/4}$

$$2 = 2B+C$$

$$2 = 2B+\frac{1}{4}$$

$$2-\frac{1}{4} = 2B$$

$$\frac{7}{4} = 2B$$

$$\boxed{B = 7/8}$$

$$X(s) = \frac{\frac{1}{8}}{s+2} + \frac{\frac{7}{8}s + \frac{1}{4}}{s^2+4}$$

$$= \frac{1/8}{s+2} + \frac{7/8\, s}{s^2+4} + \frac{1/4}{s^2+4}$$

$$= \frac{1}{8} e^{-2t} u(t) + \frac{7}{8} \cos 2t\, u(t) + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{2}{s^2+4}$$

$$= \frac{1}{8} e^{-2t} u(t) + \frac{7}{8} \cos 2t\, u(t) + \frac{1}{8} \sin 2t\, u(t)$$

Initial and final value theorems :-

If $f(t)$ is a signal then

$$f(t) \begin{cases} f(t)\big|_{t=0} \longrightarrow f(0), \text{ initial value} \\ f(t)\big|_{t=\infty} \longrightarrow f(\infty), \text{ final value} \end{cases}$$

According to initial value theorem f_t

$$f(t)\Big|_{t=0} = f(0) = \lim_{s \to \infty} sF(s)$$

Proof:

from differentiation property

$$L\left\{\frac{d}{dt} f(t)\right\} = sF(s) - f(0)$$

$$\int_0^\infty \frac{d}{dt}(f(t)) \cdot e^{-st} \cdot dt = sF(s) - f(0)$$

$$\lim_{s \to \infty} \int_0^\infty \cancelto{0}{\frac{d}{dt}(f(t)) \cdot e^{-st} \cdot dt} = \lim_{s \to \infty} sF(s) - f(0)$$

$$\boxed{f(0) = \lim_{s \to \infty} sF(s)}$$

According to the final value theorem

$$f(t)\Big|_{t=\infty} = f(\infty) = \lim_{s \to 0} sF(s)$$

Proof:-

$$L\left\{ \frac{d}{dt} f(t) \right\} = sF(s) - f(0)$$

$$\int_0^\infty \frac{d}{dt}(f(t)) \cdot e^{-st} \cdot dt = \lim_{s \to \infty} sF(s) - f(0)$$

$$\lim_{s \to 0} \int_0^\infty \frac{d}{dt}(f(t)) \cdot e^{-st} \cdot dt = \lim_{s \to 0} sF(s) - f(0)$$

Find the initial and final values of the signal

$$X(s) = \frac{5s+50}{s(s+5)}$$

Initial value $f(0) = \lim_{s\to\infty} sF(s)$

$$x(0) = \lim_{s\to\infty} \cancel{s} \cdot \frac{5s+50}{\cancel{s}(s+5)}$$

$$= \lim_{s\to\infty} \frac{5s+50}{s+5}$$

$$= \lim_{s\to\infty} \frac{5+50/s}{1+5/s}$$

$$= 5$$

Final value

$$x(\infty) = \lim_{s\to 0} s \cdot x(s)$$

$$= \lim_{s\to 0} \frac{5s+50}{s+5}$$

$$= 10$$

Note:-

final value theorem is not applicable for sinuosoidal signals unless a signal is multiplyed by a decreasing exponent (damped)

Q: Find the initial and final values of $x(s) = \frac{s}{s^2+4}$

Initial

$$x(0) = \underset{s\to\infty}{lt}\ s\,x(s)$$

$$\underset{s\to\infty}{lt}\ \frac{\cancel{s}}{\cancel{s^2}(1+4/s^2)}$$

$$= \underset{s\to\infty}{lt}\ s \cdot \frac{s}{s^2+4}$$

$$= \underset{s\to\infty}{lt}\ \frac{s^2}{s^2(1+4/s^2)}$$

$$= \underset{s\to\infty}{lt}\ \frac{1}{1+4/s^2}$$

$$= 1$$

$$x(\infty) \neq \underset{s\to 0}{lt}\ s \cdot \frac{s}{s^2+4}$$

Final value theorem is not applicable.